# Calculus

2011/11/01

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**教學進度修定：因校運動會改在12月13日，第二次期中考提前
在2011年11月29日舉行，考試範圍：4.1 − 6.1。**

**考試注意事項：**

1. 答案紙直行對折，兩直欄書寫作答。

2. 無清楚計算過程，不予計分。

3. 此次考試，禁用 L'Hôpital's rule （羅必達規則）。

**試題：**

1. (15%) Let

$$f(x) = \begin{cases} x \cot 2x & \text{if } x < 0, \\ x^2 + c & \text{if } x \geq 0. \end{cases}$$

   Find the value of $c$ that will make $f$ continuous at $x = 0$.

2. (15%) Prove that $f(x) = x^5 + 6x + 4$ has exactly one zero in $(-\infty, \infty)$.

3. Let $f(x) = x^4 - 4x^3 + 12$.

   (a) (10%) Find the relative extreme of $f(x)$.

   (b) (5%) Find the point of inflection of $f(x)$.

4. (15%) Sketch the graph of the function $f(x) = \dfrac{x^2}{x^2 + 1}$.

5. (5%, 5%) Find the limit:

$$(a) \lim_{h \to 0} \frac{(2+h)^{-1} - 2^{-2}}{h} \qquad (b) \lim_{x \to 0} \sqrt{\frac{\tan x - \sin x}{x^2}}$$

6. (10%) Find the equation of the tangent line at the point on the graph of $y = x^2 \sin 3x$, where $x = \pi/2$.

7. (10%) Find $dy/dx$ by implicit differentiation: $x + y^2 = \cos xy$.

8. (10%) Find the absolute extreme value of the function $f(x) = 3x^4 - 4x^3 - 8$ on $[-1, 2]$.