18. $f'(t) = \left(1+t^{1/2}\right)\frac{d}{dt}\left(2t^2-3\right) + \left(2t^2-3\right)\frac{d}{dt}\left(1+t^{1/2}\right) = \left(1+t^{1/2}\right)(4t) + \left(2t^2-3\right)\left(\frac{1}{2}t^{-1/2}\right)$

20. $f'(x) = \dfrac{\left(x^{1/2}+1\right)\frac{d}{dx}\left(x^{1/2}-1\right) - \left(x^{1/2}-1\right)\frac{d}{dx}\left(x^{1/2}+1\right)}{\left(x^{1/2}+1\right)^2} = \dfrac{\left(x^{1/2}+1\right)\left(\frac{1}{2}x^{-1/2}\right) - \left(x^{1/2}-1\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}+1\right)^2}$

$= \dfrac{\frac{1}{2}x^0 + \frac{1}{2}x^{-1/2} - \frac{1}{2}x^0 + \frac{1}{2}x^{-1/2}}{\left(x^{1/2}+1\right)^2} = \dfrac{x^{-1/2}}{\left(\sqrt{x}+1\right)^2} = \dfrac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}$

36. $f'(x) = \dfrac{\left(x^4-2x^2-1\right)\frac{d}{dx}(x) - x\frac{d}{dx}\left(x^4-2x^2-1\right)}{\left(x^4-2x^2-1\right)^2} = \dfrac{\left(x^4-2x^2-1\right)(1) - x\left(4x^3-4x\right)}{\left(x^4-2x^2-1\right)^2} \Rightarrow$

$f'(-1) = \dfrac{\left[(-1)^4 - 2(-1)^2 - 1\right] - (-1)\left[\left(4(-1)^3 - 4(-1)\right)\right]}{\left[(-1)^4 - 2(-1)^2 - 1\right]^2} = \dfrac{1-2-1}{(1-2-1)^2} = -\dfrac{2}{4} = -\dfrac{1}{2}$

62. a. $f(x) = 8x^7 - 6x^5 + 4x^3 - x \Rightarrow f'(x) = 56x^6 - 30x^4 + 12x^2 - 1 \Rightarrow f''(x) = 336x^5 - 120x^3 + 24x \Rightarrow$ $f'''(x) = 1680x^4 - 360x^2 + 24$, so $f'''(0) = 24$.

b. $y = x^{-1} \Rightarrow y' = -x^{-2} \Rightarrow y'' = 2x^{-3} \Rightarrow y''' = -6x^{-4}$, so $y'''\big|_{x=1} = -6(1)^{-4} = -6$.

10. $g'(x) = \dfrac{d}{dx}\left(\dfrac{\cos x}{1+x}\right) = \dfrac{(1+x)(-\sin x) - \cos x}{(1+x)^2} = -\dfrac{\sin x + \cos x + x\sin x}{(1+x)^2}$

14. $f'(x) = \dfrac{d}{dx}\left(\dfrac{\cot x}{1+\csc x}\right) = \dfrac{(1+\csc x)\left(-\csc^2 x\right) - \cot x\,(-\csc x\cot x)}{(1+\csc x)^2} = \dfrac{-\csc^2 x - \csc x\left(\csc^2 x - \cot^2 x\right)}{(1+\csc x)^2}$

$= \dfrac{-\csc x\,(\csc x + 1)}{(1+\csc x)^2} = -\dfrac{\csc x}{1+\csc x}$

24. $g'(x) = \dfrac{d}{dx}(\sec x) = \sec x \tan x \Rightarrow$

$g''(x) = \sec x\dfrac{d}{dx}(\tan x) + \tan x\dfrac{d}{dx}(\sec x) = \sec x \cdot \sec^2 x + \tan x \cdot \sec x \tan x = \sec x\left(\sec^2 x + \tan^2 x\right)$

$= \sec x\left(1 + 2\tan^2 x\right)$

41. $f(x) = \sin x \Rightarrow f'(x) = \cos x \Rightarrow f''(x) = -\sin x \Rightarrow f'''(x) = -\cos x \Rightarrow f^{(4)}(x) = \sin x \Rightarrow \cdots$. So higher order derivatives of $f(x)$ are either $\pm\sin x$ or $\pm\cos x$. Since $-1 \le \sin x \le 1$ and $-1 \le \cos x \le 1$ for all x, we see that $\left|f^{(n)}(x)\right| \le 1$ for all n and all x.