Week 15: 13.5: 5, 13, 25, 33

13.6: 3, 9, 17, 31

5. $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} = \left(6x^2y^2z\right)(1) + \left(4x^3yz\right)(-\sin t) + \left(2x^3y^2\right)(\sin t + t\cos t)$

$= 2x^2y\left[3yz - 2xz\sin t + xy(\sin t + t\cos t)\right]$

13. $\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u} = \left(\tan^{-1} yz\right)\left(\frac{1}{2\sqrt{u}}\right) + \frac{xz}{1+(yz)^2}(0) + \frac{xy}{1+(yz)^2}(-v\sin u)$

$= \frac{\left(\tan^{-1} yz\right)\sqrt{u}}{2u} - \frac{xyv\sin u}{1+y^2z^2}$ and

$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v} = \left(\tan^{-1} yz\right)(0) + \frac{xz}{1+y^2z^2}\left(-2e^{-2v}\right) + \frac{xy}{1+y^2z^2}(\cos u) = \frac{x\left(y\cos u - 2ze^{-2v}\right)}{1+y^2z^2}.$

25. $\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r} = \frac{2xy}{z^2}e^{st} + \frac{x^2}{z^2}ste^{rt} + \left(-\frac{2x^2y}{z^3}\right)ste^{rst}$ and

$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t} = \frac{2xy}{z^2}rse^{st} + \frac{x^2}{z^2}rse^{rt} + \left(-\frac{2x^2y}{z^3}\right)rse^{rst}$. If $r = 1$, $s = 2$, and $t = 0$, then $x = 1$, $y = 2$, and $z = 1$, so $\frac{\partial w}{\partial r} = 4 + 0 + (-4)(0) = 4$ and $\frac{\partial w}{\partial t} = 4(2) + 1(2) - 4(2) = 2$.

33. Here $F(x,y,z) = x^2 + xy - x^2z + yz^2 = 0$, so $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{2x+y-2xz}{-x^2+2yz} = \frac{2x+y-2xz}{x^2-2yz}$ and

$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{x+z^2}{-x^2+2yz} = \frac{x+z^2}{x^2-2yz}.$

3. Here $\mathbf{u} = \cos\frac{\pi}{2}\mathbf{i} + \sin\frac{\pi}{2}\mathbf{j} = \mathbf{j}$, so $D_{\mathbf{u}}f(3,0) = \frac{\partial f}{\partial y}(3,0) = (x+1)e^y\big|_{(3,0)} = 4.$

9. $f_x(x,y,z) = \frac{\partial}{\partial x}\left(xe^{yz}\right) = e^{yz}$, $f_y(x,y,z) = xze^{yz}$, and $f_z(x,y,z) = xye^{yz}$, so
$\nabla f(1,0,2) = \left(e^{yz}\mathbf{i} + xze^{yz}\mathbf{j} + xye^{yz}\mathbf{k}\right)\big|_{(1,0,2)} = \mathbf{i} + 2\mathbf{j}.$

17. Here $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{-2\mathbf{i}+3\mathbf{j}}{\sqrt{(-2)^2+3^2}} = -\frac{2\sqrt{13}}{13}\mathbf{i} + \frac{3\sqrt{13}}{13}\mathbf{j}$, $f_x(x,y) = \sin^2 y$, and $f_y(x,y) = 2x\sin y\cos y = x\sin 2y$, so

$D_{\mathbf{u}}f\left(-1,\frac{\pi}{4}\right) = f_x\left(-1,\frac{\pi}{4}\right)\left(-\frac{2\sqrt{13}}{13}\right) + f_y\left(-1,\frac{\pi}{4}\right)\left(\frac{3\sqrt{13}}{13}\right) = \frac{1}{2}\left(-\frac{2\sqrt{13}}{13}\right) + (-1)\left(\frac{3\sqrt{13}}{13}\right) = -\frac{4\sqrt{13}}{13}.$

31. Here $\mathbf{u} = \frac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \frac{2\mathbf{i} + \frac{\pi}{4}\mathbf{j} - \frac{\pi}{6}\mathbf{k}}{\sqrt{2^2 + \left(\frac{\pi}{4}\right)^2 + \left(\frac{\pi}{6}\right)^2}} = \frac{12}{\sqrt{576+13\pi^2}}\left(2\mathbf{i} + \frac{\pi}{4}\mathbf{j} - \frac{\pi}{6}\mathbf{k}\right)$, $f_x(x,y,z) = \sin(2y+3z)$,

$f_y(x,y,z) = 2x\cos(2y+3z)$, and $f_z(x,y,z) = 3x\cos(2y+3z)$, so

$$D_{\mathbf{u}}f\left(1,\frac{\pi}{4},-\frac{\pi}{12}\right) = f_x\left(1,\frac{\pi}{4},-\frac{\pi}{12}\right)\cdot\frac{24}{\sqrt{576+13\pi^2}} + f_y\left(1,\frac{\pi}{4},-\frac{\pi}{12}\right)\cdot\frac{3\pi}{\sqrt{576+13\pi^2}} + f_z\left(1,\frac{\pi}{4},-\frac{\pi}{12}\right)\cdot\left(-\frac{2\pi}{\sqrt{576+13\pi^2}}\right)$$

$$= \frac{\sqrt{2}}{2}\left(\frac{24}{\sqrt{576+13\pi^2}}\right) + \sqrt{2}\left(\frac{3\pi}{\sqrt{576+13\pi^2}}\right) + \frac{3\sqrt{2}}{2}\left(-\frac{2\pi}{\sqrt{576+13\pi^2}}\right) = \frac{12\sqrt{2}}{\sqrt{13\pi^2+576}}$$