Week 14: 13.1: 本節沒有作業

13.2: 3, 9, 11, 15

13.3: 13, 25, 35, 43

3. Along $y = 0$, $\lim_{(x,y)\to(0,0)} \frac{3xy}{3x^2+y^2} = \lim_{x\to 0} \frac{0}{3x^2} = \lim_{x\to 0} = 0$. Along $y = x$, $\lim_{(x,y)\to(0,0)} \frac{3xy}{3x^2+y^2} = \lim_{x\to 0} \frac{3x^2}{3x^2+x^2} = \lim_{x\to 0} \frac{3}{4} = \frac{3}{4}$. Because these two limits are not equal, the given limit does not exist.

9. Along the x-axis, $\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2} = \lim_{x\to 0} \frac{0}{x^2} = 0$. Let C denote the curve with parametric equations $x = t, y = t, z = t$. Then along C, $\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2} = \lim_{t\to 0} \frac{3t^2}{3t^2} = \lim_{t\to 0} 1 = 1$. Because these two limits are not equal, the given limit does not exist.

11. Along the z-axis, $\lim_{(x,y,z)\to(0,0,0)} \frac{xz^2+2y^2}{x^2+2y^2+z^4} = \lim_{z\to 0} \frac{0}{z^4} = \lim_{z\to 0} 0 = 0$. Let $C: x = t^2, y = t^2, z = t$. Then along C, $\lim_{(x,y,z)\to(0,0,0)} \frac{xz^2+2y^2}{x^2+2y^2+z^4} = \lim_{t\to 0} \frac{t^4+2t^4}{t^4+2t^4+t^4} = \lim_{t\to 0} \frac{3}{4} = \frac{3}{4}$. Because these two limits are not equal, the given limit does not exist.

15. $\lim_{(x,y)\to(1,2)} \frac{2x^2-3y^3+4}{3-xy} = \frac{2(1)^2-3(2)^3+4}{3-(1)(2)} = -18$

13. $f_x(x,y) = \frac{\partial}{\partial x}\left(xe^{y/x}\right) = e^{y/x} + xe^{y/x}\left(-\frac{y}{x^2}\right) = e^{y/x}\left(1-\frac{y}{x}\right)$ and $f_y(x,y) = \frac{\partial}{\partial y}\left(xe^{y/x}\right) = xe^{y/x}\left(\frac{1}{x}\right) = e^{y/x}$.

25. $g_x(x,y,z) = \frac{\partial}{\partial x}\left(x^{1/2}y^{1/2}z^{1/2}\right) = \frac{1}{2}x^{-1/2}y^{1/2}z^{1/2} = \frac{\sqrt{xyz}}{2x}$. By symmetry, $g_y(x,y,z) = \frac{\sqrt{xyz}}{2y}$ and $g_z(x,y,z) = \frac{\sqrt{xyz}}{2z}$.

35. $g_x(x,y) = \frac{\partial}{\partial x}\left(x^3y^2+xy^3-2x+3y+1\right) = 3x^2y^2+y^3-2$,
$g_y(x,y) = \frac{\partial}{\partial y}\left(x^3y^2+xy^3-2x+3y+1\right) = 2x^3y+3xy^2+3$, $g_{xx}(x,y) = \frac{\partial}{\partial x}\left(3x^2y^2+y^3-2\right) = 6xy^2$,
$g_{yy} = \frac{\partial}{\partial y}\left(2x^3y+3xy^2+3\right) = 2x^3+6xy$, $g_{xy} = \frac{\partial}{\partial y}\left(3x^2y^2+y^3-2\right) = 6x^2y+3y^2$, and
$g_{yx} = \frac{\partial}{\partial x}\left(2x^3y+3xy^2+3\right) = 6x^2y+3y^2$.

43. $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x\cos y + y\sin x) = \cos y + y\cos x \Rightarrow \frac{\partial^2 z}{\partial y\,\partial x} = \frac{\partial}{\partial y}(\cos y + y\cos x) = -\sin y + \cos x \Rightarrow$
$\frac{\partial^3 z}{\partial x\,\partial y\,\partial x} = \frac{\partial}{\partial x}(-\sin y + \cos x) = -\sin x$