Week 13: 11.5: 15, 31, 33, 43

12.1: 本節沒有作業

12.2: 3, 27, 28, 35

15. A vector parallel to L_1 is $\mathbf{v}_1 = \langle 3, 3, 1\rangle$ and a vector parallel to L_2 is $\mathbf{v}_2 = \langle 4, 6, 1\rangle$. Because $\mathbf{v}_1 \neq c\mathbf{v}_2$ for any scalar c, we see that $\mathbf{v}_1$ and $\mathbf{v}_2$ (and therefore L_1 and L_2) are not parallel. Suppose the two lines intersect at a point. Then there exist numbers t_1 and t_2 such that $\left.\begin{array}{rcl} -1+3t_1 &=& 1+4t_2 \\ -2+3t_1 &=& -2+6t_2 \\ 3+\ t_1 &=& 4+\ t_2 \end{array}\right] \Rightarrow \left.\begin{array}{rcl} 3t_1-4t_2 &=& 2 \\ 3t_1-6t_2 &=& 0 \\ t_1-t_2 &=& 1 \end{array}\right] \Rightarrow t_1 = 2$ and $t_2 = 1$, so the lines intersect at $(5, 4, 5)$.

31. Let $A\,(1, 0, -2)$, $B\,(1, 3, 2)$, and $C\,(2, 3, 0)$. Then $\overrightarrow{AB} = \langle 0, 3, 4\rangle$ and $\overrightarrow{AC} = \langle 1, 3, 2\rangle$. A normal to the required plane is $\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 4 \\ 1 & 3 & 2 \end{vmatrix} = \langle -6, 4, -3\rangle$, so an equation of the plane is $-6\,(x-1) + 4\,(y-0) - 3\,(z+2) = 0 \Leftrightarrow$ $6x - 4y + 3z = 0$.

33. Let $P\,(1, 3, 2)$. Setting $t = 0$ gives the point $Q\,(1, -1, 3)$ on the line. Also, a vector in the same direction as the line is $\mathbf{v} = \langle 1, -2, 2\rangle$, so a vector normal to the required plane is $\mathbf{n} = \overrightarrow{PQ} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 1 \\ 1 & -2 & 2 \end{vmatrix} = -6\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, and an equation of the plane is $-6\,(x-1) + 1\,(y-3) + 4\,(z-2) = 0 \Leftrightarrow 6x - y - 4z = -5$.

43. A normal to the plane $x + y + 2z = 6$ is $\mathbf{n} = \langle 1, 1, 2\rangle$ and a vector parallel to the line $L : x = 1 + t, y = 2 + t, z = -1 + t$ is $\mathbf{v} = \langle 1, 1, 1\rangle$, so the angle between the normal to the plane and the line is $\theta = \cos^{-1}\left(\frac{|\mathbf{n}\cdot\mathbf{v}|}{|\mathbf{n}|\,|\mathbf{v}|}\right) = \cos^{-1}\left(\frac{|\langle 1,1,2\rangle\cdot\langle 1,1,1\rangle|}{\sqrt{1+1+4}\sqrt{1+1+1}}\right) = \cos^{-1}\frac{4}{\sqrt{6}\sqrt{3}} \approx 19.5^\circ$. Therefore, the required angle is about $90^\circ - 19.5^\circ = 70.5^\circ$.

3. $\mathbf{r}\,(t) = \left\langle t^2 - 1, \sqrt{t^2+1}\right\rangle \Rightarrow \mathbf{r}'\,(t) = \left\langle 2t, \frac{t}{\sqrt{t^2+1}}\right\rangle$ and since

$$\frac{d}{dt}\frac{t}{\sqrt{t^2+1}} = \frac{d}{dt}\left[t\left(t^2+1\right)^{-1/2}\right] = \left(t^2+1\right)^{-1/2} + t\left(-\tfrac{1}{2}\right)\left(t^2+1\right)^{-3/2}(2t) = \frac{1}{\left(t^2+1\right)^{3/2}},$$

$\mathbf{r}''\,(t) = \left\langle 2, \frac{1}{\left(t^2+1\right)^{3/2}}\right\rangle$.

27. $\int\left(t\mathbf{i} + 2t^2\mathbf{j} + 3\mathbf{k}\right)dt = \frac{1}{2}t^2\mathbf{i} + \frac{2}{3}t^3\mathbf{j} + 3t\mathbf{k} + \mathbf{C}$

28. $\int_0^1\left(t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}\right)dt = \left(\frac{1}{2}t^2\mathbf{i} + \frac{1}{3}t^3\mathbf{j} + \frac{1}{4}t^4\mathbf{k}\right)\Big|_0^1 = \frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{4}\mathbf{k}$

35. $\mathbf{r}\,(t) = \int\mathbf{r}'\,(t)\,dt = \int\left(2\mathbf{i} + 4t\mathbf{j} - 6t^2\mathbf{k}\right)dt = 2t\mathbf{i} + 2t^2\mathbf{j} - 2t^3\mathbf{k} + \mathbf{C}$ and $\mathbf{r}\,(0) = \mathbf{i} + \mathbf{k} \Rightarrow \mathbf{C} = \mathbf{i} + \mathbf{k}$, so $\mathbf{r}\,(t) = (2t+1)\,\mathbf{i} + 2t^2\mathbf{j} - \left(2t^3 - 1\right)\mathbf{k}$.