6. $f(x) = x^3 - x \quad \Rightarrow$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\left[(x+h)^3-(x+h)\right]-\left(x^3-x\right)}{h}$$
$$= \lim_{h\to 0}\frac{\left(x^3+3x^2h+3xh^2+h^3-x-h\right)-\left(x^3-x\right)}{h} = \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3-h}{h}$$
$$= \lim_{h\to 0}\frac{h\left(3x^2+3xh+h^2-1\right)}{h} = \lim_{h\to 0}\left(3x^2+3xh+h^2-1\right) = 3x^2-1 \text{ with domain } (-\infty,\infty).$$

10. $f(x) = \dfrac{1}{x} \quad \Rightarrow$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h}$$
$$= \lim_{h\to 0}\frac{-h}{hx(x+h)} = -\lim_{h\to 0}\frac{1}{x(x+h)} = -\frac{1}{x^2} \text{ with domain } (-\infty,0)\cup(0,\infty).$$

16. $f(x) = 3x^2 - 4x + 2 \quad \Rightarrow$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\left[3(x+h)^2-4(x+h)+2\right]-\left(3x^2-4x+2\right)}{h}$$
$$= \lim_{h\to 0}\frac{\left(3x^2+6xh+3h^2-4x-4h+2\right)-\left(3x^2-4x+2\right)}{h} = \lim_{h\to 0}\frac{h(6x+3h-4)}{h}$$
$$= \lim_{h\to 0}(6x+3h-4) = 6x-4$$

The slope of the tangent line at $(2,6)$ is $f'(2) = 6(2)-4 = 8$. An equation of the tangent line is $y-6 = 8(x-2)$ or $y = 8x-10$.

50. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(x+1) = 1$, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(x^2+1\right) = 1$. Therefore, $\lim_{x\to 0} f(x) = 1$. Also, $f(0) = 0+1 = 1$, and so $\lim_{x\to 0} f(x) = f(0)$. Therefore, f is continuous at 0.

To show that f is not differentiable at 0, let $h < 0$ and consider $\lim_{h\to 0^-}\dfrac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-}\dfrac{(h+1)-1}{h} = \lim_{h\to 0^-} 1 = 1$. Next, if $h > 0$, then $\lim_{h\to 0^+}\dfrac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+}\dfrac{\left[(0+h)^2+1\right]-1}{h} = \lim_{h\to 0^+} h = 0$. This shows that $\lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$ does not exist, and so by definition, f is not differentiable at 0.

24. $f'(x) = \dfrac{d}{dx}\left[-\frac{1}{3}\left(x^{-3}-x^6\right)\right] = -\frac{1}{3}\left(-3x^{-4}-6x^5\right) = x^{-4}+2x^5 = 2x^5+\dfrac{1}{x^4}$

32. $f'(u) = \dfrac{d}{du}\left(u^{-1/2}-3u^{-1/3}\right) = -\frac{1}{2}u^{-3/2}+u^{-4/3} = -\dfrac{1}{2u^{3/2}}+\dfrac{1}{u^{4/3}}$

48. $y = \frac{1}{3}x^3-2x+5 \Rightarrow \dfrac{dy}{dx} = x^2-2$. The slope of the given line is 2, so set $x^2-2 = 2 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$. The required points are $\left(-2, \frac{19}{3}\right)$ and $\left(2, \frac{11}{3}\right)$.

50. The line $y = 2x$ has slope 2. Also $y = x^2+c \Rightarrow \dfrac{dy}{dx} = 2x$, so $2x = 2 \Rightarrow x = 1$. Therefore $y = 2$. Substituting into the second equation gives $2 = 1+c$, so $c = 1$.