Calculus II **First Midterm** Mar 30, 2010

1. (15%) Find the Maclaurin series for the function $g(x) = 2\sin x^3$.

2. (15%) Find a power series, centered at 0, for $f(x) = \frac{3x-1}{x^2-1}$. Also determine the interval of convergence of the power series.

3. (15%) Find the intervals of convergence of (a)$f(x)$, (b)$f'(x)$, (c)$\int f(x)dx$, where

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}.$$

   You must include a check for convergence at the endpoints of the interval.

4. (15%) Use Taylor's theorem to obtain an upper bound for the error of the approximation.

$$e \approx 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} + \frac{1^5}{5!}$$

5. Determine the convergence or divergence of each series.

   (a) (5%) $\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$
   (b) (5%) $\sum_{n=1}^{\infty} (\frac{\pi}{6})^n$
   (c) (5%) $\sum_{n=1}^{\infty} ne^{-n^2}$
   (d) (5%) $\sum_{n=1}^{\infty} \frac{1}{3n+1}$
   (e) (5%) $\sum_{n=1}^{\infty} (-1)^n \frac{3}{4n+1}$
   (f) (5%) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$
   (g) (5%) $\sum_{n=1}^{\infty} (\frac{n+1}{2n+1})^n$
   (h) (5%) $\sum_{n=1}^{\infty} \frac{1}{n^\pi}$