1. (15%) Find the minimum value of $f(x,y,z) = 2x^2+y^2+3z^2$ subject to the constraint $2x-3y-4z = 49$.
2. (15%) Examine the function $f(x,y) = -5x^2 + 4xy - y^2 + 16x + 10$ for relative extrema.
3. (15%) Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ by using Chain Rule where $w = x\cos yz$, $x = s^2$, $y = t^2$, $z = s - 2t$.
4. (15%) Show that $f_x(0,0)$ and $f_y(0,0)$ both exit, but that f is not differentiable at$(0,0)$ where f is defined as

$$f(x,y) = \begin{cases} \frac{-3xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

5. (10%) Find the equation of the tangent plane to the function $z^2-2x^2-2y^2 = 12$ at the point $(1,-1,4)$.
6. (10%) Find the directional derivative of the function $f(x,y) = \frac{y}{x+y}$ in the direction of $\mathbf{u} = \cos(-\frac{\pi}{6})\mathbf{i} + \sin(-\frac{\pi}{6})\mathbf{j}$.
7. (10%) Find the length of one turn of the helix given by $\mathbf{r}(t) = b\cos t\mathbf{i} + b\sin t\mathbf{j} + \sqrt{1-b^2}t\mathbf{k}$.
8. (10%) Find the area of one petal of the rose curve given by $r = 3\cos 3\theta$.