6.1

28. $g(x) = \ln\left(x^2+4\right)^2 = 2\ln\left(x^2+4\right)$

$$\Rightarrow g'(x) = 2\frac{d}{dx}\ln\left(x^2+4\right) = \frac{2(2x)}{x^2+4} = \frac{4x}{x^2+4}$$

30. $y = \sqrt{\ln x} = (\ln x)^{1/2} \Rightarrow y' = \frac{1}{2}(\ln x)^{-1/2}\cdot\frac{d}{dx}\ln x = \frac{1}{2x\sqrt{\ln x}}$

40. $f(x) = \ln(x\ln(x+2)) = \ln x + \ln(\ln(x+2))$,

so $f'(x) = \dfrac{1}{x} + \dfrac{\frac{1}{x+2}}{\ln(x+2)} = \dfrac{(x+2)\ln(x+2)+x}{x(x+2)\ln(x+2)}$.

42. $h'(t) = \dfrac{d}{dt}[t\sin(\ln 2t)] = \sin(\ln 2t) + t\dfrac{d}{dt}[\sin(\ln 2t)]$

$$= \sin(\ln 2t) + t\cos(\ln 2t)\cdot\frac{1}{t} = \sin(\ln 2t) + \cos(\ln 2t)$$

51. $\ln\frac{x}{y} + x - y^2 = 0 \Rightarrow \ln x - \ln y + x - y^2 = 0$. Differentiating,

we obtain $\dfrac{1}{x} - \dfrac{y'}{y} + 1 - 2yy' = 0 \Rightarrow y'\left(\dfrac{1}{y}+2y\right) = 1 + \dfrac{1}{x}$

$$\Rightarrow y' = \frac{(x+1)y}{x\left(2y^2+1\right)}.$$

52. $\ln(x+y) - \cos y - x^2 = 0 \Rightarrow \dfrac{1+y'}{x+y} + (\sin y)y' - 2x = 0$

$$\Rightarrow \frac{1}{x+y} - 2x + \left(\frac{1}{x+y} + \sin y\right)y' = 0$$

$$\Rightarrow \frac{1-2x(x+y)}{x+y} + \frac{1+(x+y)\sin y}{x+y}y' = 0$$

$$\Rightarrow y' = \frac{2x(x+y)-1}{1+(x+y)\sin y}$$

56. $y - \ln\left(x^2 + y^2\right) = 0 \Rightarrow y' - \dfrac{2x + 2yy'}{x^2 + y^2} = 0$. Substituting $x = 1$ and $y = 0$

into the equation gives $y' - \dfrac{2+0}{1+0} = 0$ or $y' = 2$, the slope of the required tangent line.

An equation is $y - 0 = 2(x - 1)$ or $y = 2x - 2$.

66. $y = \dfrac{x^2\sqrt{2x-4}}{(x+1)^2} \Rightarrow \ln y = 2\ln x + \frac{1}{2}\ln(2x-4) - 2\ln(x+1)$

$$\Rightarrow \frac{y'}{y} = \frac{2}{x} + \frac{1}{2x-4} - \frac{2}{x+1} = \frac{x^2+5x-8}{2x(x-2)(x+1)}$$

$$\Rightarrow y' = \frac{x\left(x^2+5x-8\right)}{\sqrt{2x-4}\,(x+1)^3}$$

69. Taking logarithms of both sides gives $\ln y = \ln x^x = x\ln x$,

so $\frac{y'}{y} = x\left(\frac{1}{x}\right) + \ln x = 1 + \ln x \Rightarrow y' = (\ln x + 1)\,x^x$.

Therefore, $y'' = (\ln x + 1)\frac{d}{dx}x^x + x^x\frac{d}{dx}(\ln x + 1)$

$$= (\ln x + 1)(\ln x + 1)\,x^x + x^x\left(\tfrac{1}{x}\right)$$

$$= \left[x(\ln x + 1)^2 + 1\right]x^{x-1}.$$

72. Let $u = 2x + 3$. Then $du = 2\,dx$,

so $\displaystyle\int \frac{1}{2x+3}\,dx = \frac{1}{2}\int\frac{du}{u} = \tfrac{1}{2}\ln|u| + C = \tfrac{1}{2}\ln|2x+3| + C$.

74. $\displaystyle\int_1^3 \frac{x^2 - x + 3}{x}\,dx = \int_1^3\left(x - 1 + \frac{3}{x}\right)dx$

$$= \left(\tfrac{1}{2}x^2 - x + 3\ln|x|\right)\Big|_1^3 = 2 + 3\ln 3$$

76. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$, $x = 1$

$\Rightarrow \quad u = 0$, and $x = 3 \Rightarrow u = \ln 3$.

Thus, $\displaystyle\int_1^3 \frac{\ln x}{x}\,dx = \int_0^{\ln 3} u\,du = \tfrac{1}{2}u^2\Big|_0^{\ln 3} = \tfrac{1}{2}(\ln 3)^2.$

78. Let $u = 1 + \ln x$. Then $du = \dfrac{dx}{x}$,

so $\displaystyle\int \frac{\sqrt{1+\ln x}}{x}\,dx = \int \sqrt{u}\,du = \tfrac{2}{3}u^{3/2} + C = \tfrac{2}{3}(1+\ln x)^{3/2} + C.$

80. Let $u = 4 - \tan 3x$. Then $du = -3\sec^2 3x\,dx$,

so $\displaystyle\int \frac{\sec^2 3x}{4-\tan 3x}\,dx = -\frac{1}{3}\int \frac{du}{u} = -\tfrac{1}{3}\ln|u| + C = -\tfrac{1}{3}\ln|4-\tan 3x| + C.$

82. Let $u = 1 + \sin^2 x$. Then $du = 2\sin x\cos x\,dx = \sin 2x\,dx$, so
$\displaystyle\int \frac{\sin 2x}{1+\sin^2 x}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln\left(1+\sin^2 x\right) + C.$

84. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$, so $\displaystyle I = \int \frac{\ln x\sqrt{1+\ln x}}{x}\,dx = \int u\,(1+u)^{1/2}\,du.$

Now let $v = 1 + u$. Then $dv = du$ and

$I = \int (v-1)\,v^{1/2}\,dv = \int \left(v^{3/2} - v^{1/2}\right)dv = \tfrac{2}{5}v^{5/2} - \tfrac{2}{3}v^{3/2} + C$

$= \tfrac{2}{15}v^{3/2}(3v-5) + C = \tfrac{2}{15}(1+u)^{3/2}(3u-2) + C$

$= \tfrac{2}{15}(\ln x + 1)^{3/2}(3\ln x - 2) + C$

91. $\frac{dy}{dx} = \frac{d}{dx}\int_x^{x^2} \ln t\,dt = \frac{d}{dx}\left[\int_x^c \ln t\,dt + \int_c^{x^2} \ln t\,dt\right] = \frac{d}{dx}\left[-\int_c^x \ln t\,dt + \int_c^{x^2} \ln t\,dt\right]$

$= -\ln x + \left(\ln x^2\right)\frac{d}{dx}\left(x^2\right) = -\ln x + 2x\ln x^2 = (4x-1)\ln x$

92. a. $y=\int_{2/x}^{x^2}\frac{dt}{t}=\ln|t|\Big|_{2/x}^{x^2}=\ln x^2-\ln\frac{2}{x}=2\ln x-\ln 2+\ln x=3\ln x-\ln 2,$

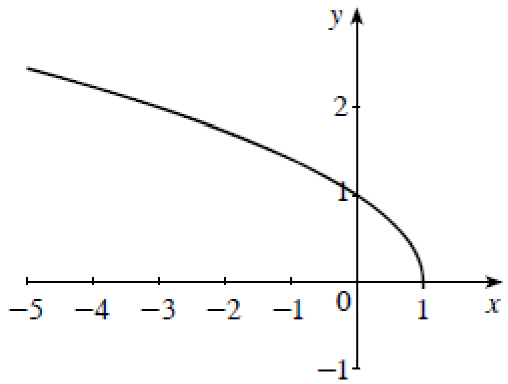
so $\frac{dy}{dx}=\frac{d}{dx}(3\ln x-\ln 2)=\frac{3}{x}.$

b. $\frac{dy}{dx}=\frac{d}{dx}\int_{2/x}^{x^2}\frac{dt}{t}=\frac{d}{dx}\left[\int_{2/x}^{c}\frac{dt}{t}+\int_{c}^{x^2}\frac{dt}{t}\right]=\frac{d}{dx}\left[-\int_{c}^{2/x}\frac{dt}{t}+\int_{c}^{x^2}\frac{dt}{t}\right]$

$=-\frac{1}{2/x}\frac{d}{dx}\left(\frac{2}{x}\right)+\frac{1}{x^2}\frac{d}{dx}\left(x^2\right)=\left(-\frac{x}{2}\right)\left(-\frac{2}{x^2}\right)+\frac{1}{x^2}(2x)=\frac{1}{x}+\frac{2}{x}=\frac{3}{x}$

6.2

15. $f(x)=\sqrt{1-x}$ is one-to-one. There is no horizontal line that cuts the graph of f at more than one point.



20. By definition, $f\left(f^{-1}(7)\right)=7.$

22. By inspection $f(0)=2$, so $f^{-1}(2)=0.$

26. By inspection $f(1)=1$, so $f^{-1}(1)=1.$

50. a. $f(x)=\frac{x+1}{2x-1}\Rightarrow f(1)=\frac{2}{1}=2$, so $(1,2)$ lies on the graph of f.

b. $f'(x)=\frac{(2x-1)-(x+1)(2)}{(2x-1)^2}=-\frac{3}{(2x-1)^2}$

$\Rightarrow g'(2)=\frac{1}{f'(g(2))}=\frac{1}{f'(1)}=-\frac{(2x-1)^2}{3}\Bigg|_{x=1}=-\frac{1}{3}$

53. **a.** $f(x) = \dfrac{1}{1+x^2} \Rightarrow f(2) = \frac{1}{5}$, so $\left(2, \frac{1}{5}\right)$ lies on the graph of f.

b. $f'(x) = -\dfrac{2x}{\left(1+x^2\right)^2} \Rightarrow g'\left(\frac{1}{5}\right) = \dfrac{1}{f'\left(g\left(\frac{1}{5}\right)\right)} = \dfrac{1}{f'(2)} = -\dfrac{\left(1+x^2\right)^2}{2x}\Bigg|_{x=2} = -\dfrac{25}{4}$

56. $H'(x) = \dfrac{d}{dx}\left[g(g(x))\right] = g'(g(x))\,g'(x)$, so

$$H'(3) = g'(g(3))\,g'(3) = g'(4)\,g'(3) = \frac{1}{f'(g(4))} \cdot \frac{1}{f'(g(3))}$$

$$= \frac{1}{f'(5)} \cdot \frac{1}{f'(4)} = \frac{1}{2} \cdot \frac{1}{\frac{1}{2}} = 1.$$

57. Observe that $f(2) = \displaystyle\int_2^2 \frac{dt}{\sqrt{1+t^3}} = 0$, showing that $(2, 0)$ lies on the graph of f. Next, observe that $f'(x) = \dfrac{d}{dx}\displaystyle\int_2^x \frac{dt}{\sqrt{1+t^3}} = \frac{1}{\sqrt{1+x^3}}$ (by the FTC, Part 1). Therefore, $\left(f^{-1}\right)'(0) = \dfrac{1}{f'\left(f^{-1}(0)\right)} = \dfrac{1}{f'(2)} = \sqrt{1+2^3} = 3$.