1. (10 %) The point $(0, 2\sqrt{3}, -2)$ is given in rectangular coordinates. Find the spherical coordinates for this point.

2. (10 %) Reparametrize the helix $\mathbf{r}(t) = \cos t\ \mathbf{i} + \sin t\ \mathbf{j} + t\ \mathbf{k}$ with respect to arc length measured from $(1, 0, 0)$ in the direction of increasing t.

3. (10 %) If $f(x, y) = 1 + 2x\sqrt{y}$,
   (a) find the gradient of f and
   (b) find the directional derivative of f at $(3, 4)$ in the direction of $\mathbf{v} =< 4, -3 >$.

4. (10 %) Find the equation of the tangent plane at the point $(1, 0, 0)$ to the surface $z + 1 = xe^y \cos z$.

5. (15 %) If
$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0), \end{cases}$$
   (a) show that $f_x(0, 0)$ exists and
   (b) show that f is not differentiable at $(0, 0)$.

6. (15 %) If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$,
   (a) find that $\frac{\partial z}{\partial r}$, $\frac{\partial z}{\partial \theta}$, and
   (b) find that $\frac{\partial^2 z}{\partial r^2}$, $\frac{\partial^2 z}{\partial \theta^2}$, and
   (c) show that
$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}.$$

7. (15 %) Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

8. (15 %) Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.