Calculus   Midterm 2   DEC, 11, 2007

NO electronic or mechanical devices which have calculating or programming function are allowed. The act of using such a device is treated as cheating.

1. (15 %) Evaluate $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{2}{n}\left(5 + \frac{2i}{n}\right)^{10}$.

2. (15%) If $F(x) = \int_1^x f(t)dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u}du$, find $F''(2)$.

3. (15%) Evaluate $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$.

4. (15%) Evaluate $\int_0^{\frac{1}{2}} \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx$.

5. Find the volume of the solid obtained by rotating about the $y-$axis the region between $y = x$ and $y = x^2$ by

   (a) (5%) the method of cross-section (disk method)

   (b) (5%) the method of cylindrical shell   (shell method)

6. (10%) If $f(x) = 3 + x^2 + \tan(\frac{\pi x}{2})$, $-1 < x < 1$, find $(f^{-1})'(3)$.

7. (10%) Evaluate $\int \frac{e^x}{e^x+1}dx$.

8. (10 %) Evaluate $\lim\limits_{x\to\infty} \left(\frac{x}{x+1}\right)^x$.