Calculus Midterm 1 OCT 30, 2007

(1) Do not use the L'Hôpital's Rule to compute limits, otherwise you will get nothing even though your answer is correct.

(2) NO electronic or mechanical devices which have calculating or programming function are allowed. The act of using such a device is treated as cheating.

1. (20%) Find the limit.
   (a) $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$
   (b) $\lim_{x\to 1} \frac{\sqrt{x}-x^2}{1-\sqrt{x}}$
   (c) $\lim_{x\to 0} \frac{\sin 7x}{4x}$
   (d) Prove that $\lim_{x\to 0} \frac{|x|}{x}$ does not exist.
2. (10%) Show that $f'(0) = 0$, where
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$
   (Hint: use the squeeze theorem to find $\lim_{x\to 0} x \sin \frac{1}{x}$)
3. (15%) Find an equation of the tangent line to the curve $y = \sin(\sin x)$ at the given point $(\pi, 0)$.
4. (10%) Find y' and y'' if $x^4 + y^4 = 16$.
5. (10%) Find the linearization of the function $f(x) = \sqrt{x+3}$ at $x = 1$ and use it to approximate the number $\sqrt{4.05}$.
6. (10%) Show that the equation $x^3 - 15x + c = 0$ has at most one root in the interval $[-2, 2]$.
7. (10%) Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ that has a local maximum value of 3 at -2 and a local minimum value of 0 at 1.
8. (15%) Sketch the curve $y = \frac{2x^2}{x^2-1}$.
   (please provide the following information at least: **domain, asymptotes, intervals of increase or decrease, local maximum and minimum values, concavity and points of inflection**)