1.4

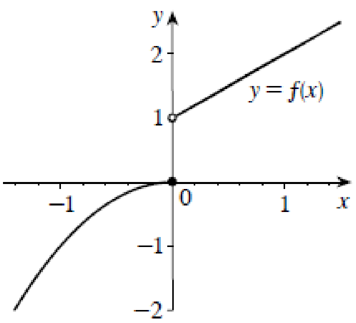
17. f is discontinuous at $0, \pm 1, \pm 2, \ldots$.

27. We require that $f(1) = 1 + 2 = 3 = \lim_{x \to 1^+} kx^2 = k$, or $k = 3$.

54. $\lim_{x \to 4} \frac{4 - x}{2 - \sqrt{x}} = \lim_{x \to 4} \frac{(2 - \sqrt{x})(2 + \sqrt{x})}{2 - \sqrt{x}} = \lim_{x \to 4} (2 + \sqrt{x}) = 4$. So we define $f(x) = \begin{cases} \frac{4 - x}{2 - \sqrt{x}} & \text{if } x \neq 4 \\ 4 & \text{if } x = 4 \end{cases}$

66. $f(x) = x^4 - 2x^3 - \sqrt{x - 1}$ is continuous on $[2, 3]$. $f(2) = -1 < 0$ and $f(3) = 27 - \sqrt{2} > 0$. Therefore, by Theorem 7, $f(x) = 0$ has at least one root in $(2, 3)$.

70. **a.** $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left(-x^2\right) = 0$ and $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x + 1) = 1$. This shows that f is not continuous at 0, and therefore is not continuous on $[-1, 1]$.



b. $f(-1) = -1$ and $f(1) = 2$. The number $\frac{1}{2}$ lies between $f(-1)$ and $f(1)$, but there is no c in $[-1, 1]$ such that $f(c) = \frac{1}{2}$. (See the figure.)

1.5

10. a. $m_{\sec} = \dfrac{f(2+h) - f(2)}{(2+h) - 2} = \dfrac{\left[(2+h)^2 - (2+h)\right] - \left(2^2 - 2\right)}{h} = \dfrac{h^2 + 3h}{h} = 3 + h$

b. $m_{\tan} = \lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{(2+h) - 2} = \lim\limits_{h \to 0} (3 + h) = 3$

c. $y - 2 = 3(x - 2) \Rightarrow y = 3x - 4$

13. a. $m_{\sec} = \dfrac{f(1+h) - f(1)}{(1+h) - 1} = \dfrac{\dfrac{1}{1+h} - \dfrac{1}{1}}{h} = \dfrac{1 - (1+h)}{h(1+h)} = \dfrac{-h}{h(1+h)} = -\dfrac{1}{1+h}$

b. $m_{\tan} = \lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{(1+h) - 1} = \lim\limits_{h \to 0} \left(-\dfrac{1}{1+h}\right) = -1$

c. $y - 1 = -1(x - 1) \Rightarrow y = -x + 2$

16. $\lim\limits_{h \to 0} \dfrac{g(-1+h) - g(-1)}{(-1+h) - (-1)} = \lim\limits_{h \to 0} \dfrac{\left[(-1+h)^2 - (-1+h) + 2\right] - \left[(-1)^2 - (-1) + 2\right]}{h}$

$= \lim\limits_{h \to 0} \dfrac{h^2 - 2h + 1 - h + 1 + 2 - 4}{h} = \lim\limits_{h \to 0} \dfrac{h(h-3)}{h} = -3$

19. $\lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{(1+h) - 1} = \lim\limits_{h \to 0} \dfrac{\left[\dfrac{2}{1+h} + (1+h)\right] - \left(\dfrac{2}{1} + 1\right)}{h} = \lim\limits_{h \to 0} \dfrac{\dfrac{2}{1+h} + h - 2}{h}$

$= \lim\limits_{h \to 0} \dfrac{2 + h + h^2 - 2 - 2h}{h(1+h)} = \lim\limits_{h \to 0} \dfrac{h(h-1)}{h(1+h)} = -1$

27. The area function for a circle is $A(r) = \pi r^2$.

a. $A(1) = \pi$ and $A(2) = 4\pi$, so the average rate of change over $[1, 2]$ is $\dfrac{A(2) - A(1)}{2 - 1} = \dfrac{4\pi - \pi}{1} = 3\pi$ units^2/unit.

b. $\lim\limits_{h \to 0} \dfrac{A(2+h) - A(2)}{(2+h) - 2} = \lim\limits_{h \to 0} \dfrac{\pi(2+h)^2 - \pi(2)^2}{h} = \pi \lim\limits_{h \to 0} \dfrac{4 + 4h + h^2 - 4}{h} = \pi \lim\limits_{h \to 0} \dfrac{h(4+h)}{h} = 4\pi$ units^2/unit