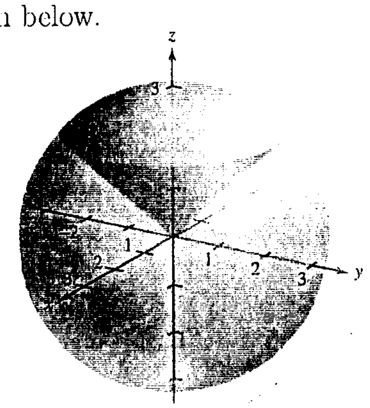
1. (15 points) Evaluate

$$\int_0^1 \int_y^1 e^{-x^2}\,dxdy.$$

2. (15 points) Find the volume of the solid region Q bounded below by the upper nappe of the cone $z^2 = x^2 + y^2$ and above by the sphere $x^2 + y^2 + z^2 = 9$, as shown below.



3. (15 points) Let R be the region bounded by the lines

$$x - 2y = 0, \quad x - 2y = -4, \quad x + y = 4, \quad \text{and} \quad x + y = 1.$$

Evaluate the double integral

$$\iint_R 3xy\,dA.$$

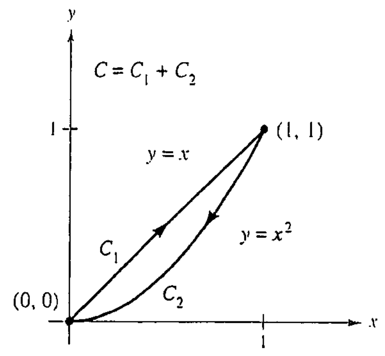
4. (15 points) Let $\mathbf{F}(x, y, z) = M\mathbf{i}+N\mathbf{j}+P\mathbf{k}$ be a vector field and M, N, and P have continuous second partial derivatives. Show that

$$\text{div}(\textbf{curl }\mathbf{F}) = 0.$$

5. (10 points) Evaluate

$$\int_C x\,ds,$$

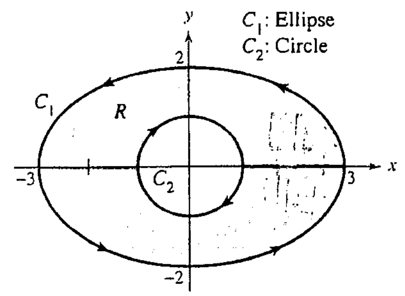
where C is the piecewise smooth curve shown below.



6. (10 points) Let R be the region inside the ellipse $(x^2/9) + (y^2/4) = 1$ and outside the circle $x^2 + y^2 = 1$. Evaluate the line integral

$$\int_C 2xy\,dx + \left(x^2 + 2x\right)dy$$

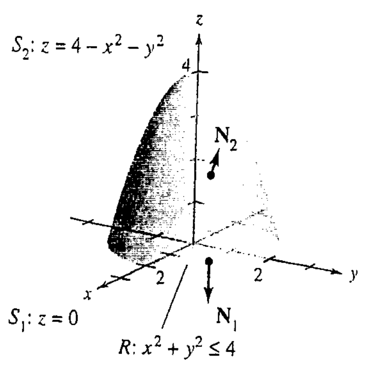
where $C = C_1 + C_2$ is the boundary of R, as shown below.



7. (10 points) Let Q be the solid region between the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane. Use the Divergence Theorem to evaluate

$$\int_S\int \mathbf{F} \cdot \mathbf{N} dS,$$

where $\mathbf{F}(x, y, z) = 2z\mathbf{i} + x\mathbf{j} + y^2\mathbf{k}$ and S is the boundary of Q, as shown below.



8. (10 points) Let C be the oriented triangle lying in the plane $2x+2y+z = 6$, as shown below. Using the Stoke's Theorem to evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F}(x, y, z) = -y^2\mathbf{i}+z\mathbf{j}+x\mathbf{k}$.