Calculus Midterm 2 May 15, 2007

No electronic or mechanical devices which have calculating or programming function.

1. (15 points) Find the area of the surface formed by revolving the circle $r = f(\theta) = \cos\theta$ about the line $\theta = \pi/2$.

2. (15 points) Use the following function to prove that (a) $f_x(0.0)$ and $f_y(0.0)$ exist, and (b) f is not differentiable at $(0,0)$.

$$f(x,y) = \begin{cases} \frac{5x^2y}{x^3+y^3} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

3. (15 points) Find the directional derivative of $f(x,y) = 3x^2 - 2y^2$ at $\left(-\frac{3}{4}, 0\right)$ in the direction from $P\left(-\frac{3}{4}, 0\right)$ to $Q(0,1)$.

4. (15 points) Find the extreme values of $f(x,y) = x^2 + 2y^2 - 2x + 3$ subject to the constraint $x^2 + y^2 \leq 10$.

5. (10 points) Find the principal unit normal vector for the helix given by $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + t\,\mathbf{k}$.

6. (10 points) Show that the curvature of a circle of radius r is $K = 1/r$.

7. (10 points) For $w = x\cos yz$, find $\partial w/\partial s$ and $\partial w/\partial t$ by using the appropriate Chain Rule, where $x = s^2, y = t^2$ and $z = s - 2t$.

8. (10 points) Find an equation of the tangent plane to the surface at the given point: $z = e^x(\sin y + 1), \left(0, \frac{\pi}{2}, 2\right)$.