

No electronic or mechanical devices which have calculating or programming function.

1. (5 points each) Determine the convergence or divergence of each series.

$$(a) \sum_{n=1}^{\infty} \left(1 + \frac{k}{n}\right)^n \quad (b) \sum_{n=2}^{\infty} \frac{1}{n \ln(n^2)} \quad (c) \sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)^2}$$
$$(d) \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \quad (e) \sum_{n=1}^{\infty} \frac{n!}{n3^n} \quad (f) \sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$$

2. (10 points) Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{2}{4n^2 - 1}.$$

3. (15 points) Find the fourth Taylor polynomial for $f(x) = \ln x$ centered at $c = 1$.
4. (15 points) Find the intervals of convergence of $f'(x)$ and $\int f(x)dx$, where

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}.$$

5. (15 points) Determine a power series, centered at 0, for the function

$$\frac{-2}{x^2 - 1}.$$

Identify the interval of convergence.

6. (15 points) Find a Maclaurin series for the function

$$\int_0^x (e^{-t^2} - 1) dt.$$