

考試注意事項：

1. 答案紙直行對折，兩直攔書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (15%) Find the relative extrema of $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.
2. (15%) Find the extrema of the function subject to the **inequality constraint**:

$$f(x, y) = 3x^2 + 2y^2 - 2x - 1; \quad x^2 + y^2 \leq 9.$$

3. (15%) Evaluate the integral by reversing the order of integration:

$$\int_0^1 \int_{2y}^2 e^{-x^2} dx dy.$$

4. (15%) Evaluate the integral by making a suitable change of variables:

$$\iint_R e^{(x+y)/(x-y)} dA,$$

where R is the trapezoidal region with vertices $(-2, 0)$, $(-1, 0)$, $(0, 1)$, and $(0, 2)$.

5. (10%) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ does not exist.
6. (5%,5%) Let $w = x^2y - xy^3$, where $x = \cos t$ and $y = e^t$. Use the **chain rule** to find dw/dt and its value when $t = 0$.
7. (5%,5%) Let $f(x, y) = x^2 - 2xy$.
 - (a) Find the gradient of f at the point $(1, -2)$.
 - (b) Use the result of (a) to find the directional derivative of f at $(1, -2)$ in the direction from $P(-1, 2)$ to $Q(2, 3)$
8. (10%) Find the equation of the tangent plane to the ellipsoid with equation $4x^2 + y^2 + 4z^2 = 16$ at the point $(1, 2, \sqrt{2})$.