

# 微積分七系共同教學考題

九十四學年度微積分下學期第二次期中考

- 前四題每題十五分, 後四題每題十分。
- 將桌面淨空, 並準備學生證以備查驗。
- 將答案卷對摺, 每頁兩欄書寫(two columns)
- 不可使用含有計算功能之電子儀器設備, 每題作答須有計算或推導過程, 答案卷必須寫上姓名學號科系, 否則一律以零分計。

1. Find an equation of the tangent plane to the hyperboloid given by

$$z^2 - 2x^2 - 2y^2 = 12$$

at the point  $(1, -1, 4)$ .

2. Find the  $\frac{\partial w}{\partial s}$  and  $\frac{\partial w}{\partial t}$  when  $s = 1$  and  $t = 2\pi$  for the function given by

$$w = xy + yz + xz \text{ where } x = s \cos t, y = s \sin t, \text{ and } z = t.$$

3. Find the curvature of the curve given by

$$\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} - \frac{1}{3}t^3\mathbf{k}.$$

4. Show that  $f_x(0,0)$  and  $f_y(0,0)$  both exist, but that  $f$  is not differentiable at  $(0,0)$  where  $f$  is defined as

$$f(x, y) = \begin{cases} \frac{-3xy}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

5. Find the principal unit normal vector for the helix given by

$$\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + t \mathbf{k}.$$

6. Find the length of the arc from  $\theta = 0$  to  $\theta = 2\pi$  for the cardioid

$$r = f(\theta) = 2 - 2 \cos \theta.$$

7. Find the area of one petal of the rose curve given by  $r = 3 \cos 3\theta$ .

8. For the curve given by

$$x = \sqrt{t} \quad \text{and} \quad y = \frac{1}{4}(t^2 - 4), \quad t \geq 0$$

find the slope and concavity at the point  $(2, 3)$ .