

微積分八系共同教學考題

九十三學年度微積分下學期第二次期中考

- 前四題每題十五分, 後四題每題十分。
- 將桌面淨空, 並準備學生證以備查驗。
- 將答案卷對摺, 每頁兩欄書寫(two columns)
- 不可使用含有計算功能之電子儀器設備, 每題作答須有計算或推導過程, 答案卷必須寫上姓名學號科系, 否則一律以零分計。

1. Let f be a differentiable function and consider the surface $z = xf(y/x)$.
Show that the tangent plane at any point $P(x_0, y_0, z_0)$ on the surface passes through the origin.
2. Use the following function to prove that (a) $f_x(0,0)$ exists, and (b) f is not differentiable at $(0,0)$.

$$f(x, y) = \begin{cases} \frac{3x^2y}{x^4+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

3. Find the directional derivative of $f(x, y) = 3x^2 - 2y^2$ at $(-\frac{3}{4}, 0)$ in the direction from $P(-\frac{3}{4}, 0)$ to $Q(0, 1)$.

4. Two objects are traveling in elliptical paths given by the following parametric equations.

$$x_1 = 4 \cos t \quad \text{and} \quad y_1 = 2 \sin t \quad \text{First object}$$

$$x_2 = 2 \sin 2t \quad \text{and} \quad y_2 = 3 \cos 2t \quad \text{Second object}$$

At what rate is the distance between the two objects changing when $t = \pi$?

5. Let

$$f(x, y) = \int_x^y \sqrt{1 + t^3} dt.$$

Find $f_x(x, y)$ and $f_y(x, y)$.

6. Find the length of one turn of the helix given by

$$\mathbf{r}(t) = b \cos t \mathbf{i} + b \sin t \mathbf{j} + \sqrt{1 - b^2} t \mathbf{k}.$$

7. Find the unit tangent vector and the principal unit normal vector for the helix given by

$$\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + t \mathbf{k}.$$

8. Find an equation in spherical coordinates for the surface represented by the rectangular equation

$$x^2 + y^2 + z^2 - 4z = 0.$$