微積分八系共同教學考題

九十三學年度微積分下學期第一次期中考

- 前四題每題十五分, 後四題每題十分。
- 將桌面淨空, 並準備學生證以備查驗。
- 將答案卷對摺, 每頁兩欄書寫(two columns)
- 不可使用含有計算功能之電子儀器設備, 每題作答須有計算或推導過程, 答案卷必須寫上姓名學號科系, 否則一律以零分計。

1. Find the distance between the point $Q(3, -1, 4)$ and the line given by $x = -2 + 3t$, $y = -2t$, and $z = 1 + 4t$.

2. Find the length of the arc from $\theta = 0$ to $\theta = 2\pi$ for the cardioid

$$r = f(\theta) = 2 - 2\cos\theta.$$

3. The prolate cycloid given by

$$x = 2t - \pi\sin t \text{ and } y = 2 - \pi\cos t$$

crosses itself at the point $(0, 2)$. Find the equations of both tangent lines at this point.

4. Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2}.$$

5. Find a power series for $f(x) = \ln x$, centered at 1.

6. Find a power series for $g(x) = \arctan x$, centered at 0.

7. Determine whether the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}.$$

8. Determine whether the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}.$$