

# 微積分八系共同教學考題

九十三年學年度微積分上學期期末考

- 前四題每題十五分, 後四題每題十分。
- 將桌面淨空, 並準備學生證以備查驗。
- 將答案卷對摺, 每頁兩欄書寫(two columns)
- 本次考試計算瑕積分時必須將取極限過程明白表示, 否則以零分計。
- 不可使用含有計算功能之電子儀器設備, 每題作答須有計算或推導過程, 答案卷必須寫上姓名學號科系, 否則一律以零分計。

1. (15%) The Gamma Function  $\Gamma(n)$  is defined by

$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx, \quad n > 0.$$

(a) Find  $\Gamma(1)$  and  $\Gamma(3)$ .

(b) Use integration by parts to show that  $\Gamma(n+1) = n\Gamma(n)$ .

2. (15%) Evaluate

$$\int_{-1}^2 \frac{dx}{x^3}.$$

3. (15%) Evaluate

$$\int \frac{dx}{\sqrt{4x^2 + 1}}.$$

4. (15%) Evaluate

$$\int \frac{1}{x^2 + 5x + 6} dx.$$

5. (10%) Evaluate

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x.$$

6. (10%) Find the volume of the solid of revolution formed by revolving the region bounded by

$$y = x - x^3$$

and the  $x$ -axis ( $0 \leq x \leq 1$ ) about the  $y$ -axis.

7. (10%) Find the arc length of the graph of

$$y = x^3/6 + 1/(2x)$$

on the interval  $[\frac{1}{2}, 2]$ .

8. (10%) The solid formed by revolving (about the  $x$ -axis) the unbounded region lying between the graph of  $f(x) = 1/x$  and the  $x$ -axis ( $x \geq 1$ ) is called Gabriel's Horn. Show that this solid has a finite volume and an infinite surface area.