微積分八系共同教學考題

九十三學年度微積分上學期第一次期中考

- 前四題每題十五分, 後四題每題十分。

- 將桌面淨空, 並準備學生證以備查驗。

- 將答案卷對摺, 每頁兩欄書寫(two columns)

- 本次考試計算極限值時, 不可使用羅必達法則。

- 不可使用含有計算功能之電子儀器設備, 每題作答須有計算或推導過程, 答案卷必須寫上姓名學號科系, 否則一律以零分計。

1. Find the limit (if it exists).

   (a)
   $$\lim_{x \to 0} \frac{\sqrt{x+5} - \sqrt{5}}{x}$$

   (b)
   $$\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$$

   (c)
   $$\lim_{x \to \infty} \frac{\cos x}{x}$$

2. Let

$$f(x) = \begin{cases} x\sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2\sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

(a) Show that f is continuous at 0.

(b) Show that f is not differentiable at 0.

(c) Show that g is differentiable at 0, and find $g'(0)$.

3. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$.

4. Analyze and sketch the graph of $f(x) = \frac{2(x^2-9)}{x^2-4}$.

5. Find the relative extrema of $f(x) = (x^2 - 4)^{2/3}$.

6. Determine the slope of the graph of

$$3(x^2 + y^2)^2 = 100xy$$

at the point $(3, 1)$.

7. Prove that $|\cos x - \cos y| \leq |x - y|$ for all x and y.

8. Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minite. Find the rate of change of the radius when the radius is 2 feet.