微積分五系共同教學考題

九十二學年度微積分下學期期末考

- 前四題每題十五分 後四題每題十分

- 每題作答須有計算或推導過程 否則以零分計

- 答案卷務必寫上姓名學號科系 否則以零分計

- 禁止使用含有計算功能之電子儀器設備 否則以零分計

- 請將答案卷對摺 單頁兩欄書寫 (two columns)

1. Let R be the region bounded by the lines $x - 2y = 0$, $x - 2y = -4$, $x + y = 4$, $x + y = 1$. Evaluate the double integral

$$\int_R \int 3xy \, dA.$$

2. Find the volume of the solid region Q bounded below by the upper nappe of the cone $z^2 = x^2 + y^2$ and above by the sphere $x^2 + y^2 + z^2 = 9$.

3. Evaluate the integral

$$\int_0^1 \int_y^1 e^{-x^2} dx dy.$$

(Note that it is necessary to switch the order of integration.)

4. Find the relative extrema of $f(x,y) = -x^3 + 4xy - 2y^2 + 1$.

5. Let R be the annular region lying between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 5$. Evaluate the integral

$$\int_R \int (x^2 + y) \, dA.$$

6. Find the minimum value of $f(x,y,z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$.

7. A rectangular box is resting on the xy-plane with one vertex at the origin. The opposite vertex lies in the plane $6x + 4y + 3z = 24$ and in the first octant. Find the maximum volume of such a box.

8. Find the surface area of the portion of the plane $z = 2 - x - y$ that lies above the circle $x^2 + y^2 \leq 1$ in the first quadrant.