微積分五系共同教學考題

九十二學年度微積分下學期第二次期中考

- 前四題每題十五分 後四題每題十分
- 每題作答須有計算或推導過程 否則以零分計
- 答案卷務必寫上姓名學號科系 否則以零分計
- 禁止使用含有計算功能之電子儀器設備 否則以零分計
- 請將答案卷對摺 單頁兩欄書寫 (two columns)

1. Let f be a differentiable function and consider the surface $z = xf(y/x)$. Show that the tangent plane at any point $P(x_0, y_0, z_0)$ on the surface passes through the origin.

2. Find the curvature of the curve given by $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} - \frac{1}{3}t^3\mathbf{k}$.

3. Find $\partial z/\partial x$ and $\partial z/\partial y$, given $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0$.

4. Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2 - y^2}{x^2 + y^2}\right)^2$$

5. Use the Chain Rule to find $\partial w/\partial s$ and $\partial w/\partial t$ for $w = 2xy$ where $x = s^2+t^2$ and $y = s/t$.

6. Find the directional derivative of $f(x, y) = x^2 \sin 2y$ at $(1, \pi/2)$ in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

7. Find the length of one turn of the helix given by $\mathbf{r}(t) = b\cos t\mathbf{i} + b\sin t\mathbf{j} + \sqrt{1-b^2}t\mathbf{k}$, $t \in [0, 2\pi]$.

8. Given $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, find each of the following.

   (a) $\mathbf{u} \times \mathbf{v}$

   (b) $\mathbf{v} \times \mathbf{u}$

   (c) $\mathbf{v} \times \mathbf{v}$