微積分五系共同教學考題

九十二學年度微積分下學期第一次期中考

- 前四題每題十五分 後四題每題十分
- 每題作答須有計算或推導過程 否則以零分計
- 答案卷務必寫上姓名學號科系 否則以零分計
- 禁止使用含有計算功能之電子儀器設備 否則以零分計
- 請將答案卷對摺 單頁兩欄書寫 (two columns)

1. Consider the function given by

$$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n} = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots .$$

   Find the intervals of convergence for each of the following.

   (a) $\int f(x)\, dx$

   (b) $f(x)$

   (c) $f'(x)$

2. Find the area of the surface formed by revolving the circle $r = f(\theta) = \cos\theta$ about the line $\theta = \pi/2$.

3. Determine the convergence or divergence of each series.

   (a)
   $$\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$$

   (b)
   $$\sum_{n=1}^{\infty} ne^{-n^2}$$

   (c)
   $$\sum_{n=1}^{\infty} \frac{1}{3n+1}$$

4. Determine the convergence or divergence of each series.

   (a)
   $$\sum_{n=1}^{\infty} (-1)^n \frac{3}{4n+1}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{n!}{10^n}$$

   (c)
   $$\sum_{n=1}^{\infty} (\frac{n+1}{2n+1})^n$$

5. For the curve given by $x = \sqrt{t}$ and $y = \frac{1}{4}(t^2 - 4)$, $t \geq 0$, find the slope at the point $(2, 3)$.

6. Find the arc length of the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ on the interval $[0, 2\pi]$.

7. Find the Maclaurin series for $f(x) = \cos\sqrt{x}$.

8. Apply the Integral Test to the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.