

微積分四系共同教學考題

九十二學年度微積分上學期第一次期中考

- 前四題為每題十五分 後四題每題十分
- 本次考試計算極限值時不可使用羅必達法則
- 每題作答須有計算或推導過程 否則以零分計
- 答案卷務必寫上姓名學號科系 否則以零分計
- 不可使用含有計算功能之電子儀器設備 否則以零分計
- 請將答案卷對摺 單頁兩欄書寫 (two columns)

1. Let

$$f(x) = \begin{cases} x \sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

Show that f is continuous, but not differentiable, at $x = 0$. Show that g is differentiable at 0, and find $g'(0)$.

2. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$.
3. Analyze and sketch the graph of $f(x) = \frac{2(x^2-9)}{x^2-4}$.
4. Find the extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.
5. Determine the points of inflection and discuss the concavity of the graph of $f(x) = x^4 - 4x^3$.
6. Suppose x and y are both differentiable functions of t and are related by the equation $y = x^2 + 3$. Find dy/dt when $x = 1$, given that $dx/dt = 2$ when $x = 1$.
7. Prove that $|\cos x - \cos y| \leq |x - y|$ for all x and y .
8. Find the limit (if it exists).

(a)

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+5} - \sqrt{5}}{x}$$

(b)

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$$

PS: 作業請勿抄襲, 以後一經確認, 作業成績以零分計。