

微積分四系共同教學考題

九十一學年度微積分下學期期末考

- 每題作答須有計算或推導過程 否則以零分計
- 答案卷務必寫上姓名學號科系 否則以零分計
- 不可使用含有計算功能之電子儀器設備 否則以零分計

1. (10%) Find $\nabla f(x, y, z)$ for the function given by $f(x, y, z) = x^2 + y^2 - 4z$ and find the direction of maximum increasing of f at the point $(2, -1, 1)$.
2. (10%) Find the minimum value of $f(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$.
3. (10%) Use polar coordinates to find the volume of the solid region bounded above by the hemisphere $z = \sqrt{16 - x^2 - y^2}$ and below by the circular region R in the xy -plane given by $x^2 + y^2 \leq 4$.
4. (10%) Find the surface area of the portion of the plane $z = 2 - x - y$ that lies above the circle $x^2 + y^2 \leq 1$ in the first quadrant.
5. (15%) Find the absolute extrema of the function $f(x, y) = \sin xy$ on the closed region given by $0 \leq x \leq \pi$ and $0 \leq y \leq 1$.
6. (15%) Evaluate

$$\int_0^{\sqrt{\pi/2}} \int_x^{\sqrt{\pi/2}} \int_1^3 \sin y^2 \, dz \, dy \, dx.$$

7. (15%) Find the volume of the solid region Q bounded below by the upper nappe of the cone $z^2 = x^2 + y^2$ and above by the sphere $x^2 + y^2 + z^2 = 9$.
8. (15%) Let R be the region bounded by the lines $x - 2y = 0$, $x - 2y = -4$, $x + y = 4$ and $x + y = 1$. Evaluate the double integral

$$\int_R \int 3xy \, dA.$$