微積分四系共同教學考題

九十一學年度微積分下學期第二次期中考

- 每題作答須有計算或推導過程 否則以零分計
- 答案卷務必寫上姓名學號科系 否則以零分計
- 不可使用含有計算功能之電子儀器設備 否則以零分計

1. (10%) Find the volume of the parallelepiped having $\mathbf{u} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{j} - 2\mathbf{k}$, and $\mathbf{w} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ as adjacent edges.

2. (10%) Find the curvature of the curve given by $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} - \frac{1}{3}t^3\mathbf{k}$.

3. (10%) Find a set of parametric equations for the tangent line to the curve of intersection of the surfaces

$$\begin{aligned} x^2 + 2y^2 + 2z^2 &= 20 \\ x^2 + y^2 + z &= 4 \end{aligned}$$

   at the point $(0, 1, 3)$.

4. (10%) Find the directional derivative of

$$f(x, y) = x^2 \sin 2y$$

   at $(1, \pi/2)$ in the direction of

$$\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}.$$

5. (15%) Find the unit tangent vector $\mathbf{T}(t)$, find the principal unit normal vector $\mathbf{N}(t)$ and find a set of parametric equations for the tangent line to the helix given by

$$\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$$

   at the point corresponding to $t = \pi/4$.

6. (15%) Two objects are traveling in elliptical paths given by the following parametric equations.

$$\begin{aligned} x_1 &= 4\cos t \quad \text{and} \quad y_1 = 2\sin t \\ x_2 &= 2\sin 2t \quad \text{and} \quad y_2 = 3\cos 2t \end{aligned}$$

   At what rate is the distance between the two objects changing when $t = \pi$.

7. (15%) Show that $f_x(0,0)$ and $f_y(0,0)$ both exist, but that f is not differentiable at $(0,0)$ where f is defined as

$$f(x,y) = \begin{cases} \frac{-3xy}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

8. (15%) Consider the function defined by

$$f(x,y) = \begin{cases} \frac{xy(x^2+y^2)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

(a) Find $f_x(x,y)$ and $f_y(x,y)$ for $(x,y) \neq (0,0)$.

(b) Use the definition of partial derivatives to find $f_x(0,0)$ and $f_y(0,0)$.

(c) Use the definition of partial derivatives to find $f_{xy}(0,0)$ and $f_{yx}(0,0)$.