微積分四系共同教學考題

九十一學年度微積分下學期第一次期中考

- 每題作答須有計算或推導過程 否則以零分計
- 答案卷務必寫上姓名學號科系 否則以零分計
- 不可使用含有計算功能之電子儀器設備 否則以零分計

1. (20%) Test for convergence or divergence using any appropriate test. Identify the test used.

   (a)
   $$\sum_{n=1}^{\infty} \frac{\sin(n^2)}{2^n}$$

   (b)
   $$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)(\ln(\ln n))}$$

   (c)
   $$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$$

   (d)
   $$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

   (e)
   $$\sum_{n=1}^{\infty} 2^{-n^2}$$

2. (10%) Find the focus of the parabola given by $y = -\frac{1}{2}x^2 - x - \frac{1}{2}$.

3. (10%) Find the Maclaurin polynomial P_2 for $f(x) = \cos x$. Use $P_2(x)$ to approximate the value of $\cos(0.1)$.

4. (15%) Consider the function given by

$$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}.$$

Find the intervals of convergence for each of the following.

(a) $\int f(x)dx$

(b) $f(x)$

(c) $f'(x)$

5. (15%) A circle of radius 1 rolls around the circumference of a larger circle of radius 4. The epicycloid traced by a point on the circumference of the smaller circle is given by

$$x = 5\cos t - \cos 5t$$

and

$$y = 5\sin t - \sin 5t.$$

Find the distance traveled by the point in one complete trip about the larger circle.

6. (15%) Find the length of the arc from $\theta = 0$ to $\theta = 2\pi$ for the cardioid

$$r = f(\theta) = 2 - 2\cos\theta.$$

7. (15%) The prolate cycloid given by

$$x = 2t - \pi\sin t$$

and

$$y = 2 - \pi\cos t$$

crosses itself at the point $(0, 2)$. Find the equations of both tangent lines at this point.