

# 微積分四系共同教學考題

九十一學年度微積分上學期期末考

- 每題作答須有計算或推導過程 否則以零分計
- 答案卷務必寫上姓名學號科系 否則以零分計
- 不可使用含有計算功能之電子儀器設備 否則以零分計

1. (15%) Let

$$I_n = \int_0^{\infty} \frac{x^{2n-1}}{(x^2+1)^{n+3}} dx,$$

$n \geq 1$ . Prove that

$$I_n = \frac{n-1}{n+2} I_{n-1}.$$

2. (15%) Find the perimeter of the hypocycloid of four cusps

$$x^{2/3} + y^{2/3} = 4.$$

3. (15%) Evaluate

$$\int_0^{\pi/2} \frac{1}{1 + \sin x + \cos x} dx.$$

4. (15%) Find the center of mass of the lamina of uniform density  $\rho$  bounded by the graph  $f(x) = 4 - x^2$  and the  $x$ -axis.

5. (10%) Evaluate

$$\lim_{x \rightarrow 1^+} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right).$$

6. (10%) Evaluate

$$\int \frac{x}{1 + e^{-x^2}} dx.$$

7. (10%) Find the area of the region bounded by the graph of  $y = x^2 + 2$ ,  $y = -x$ ,  $x = 0$ , and  $x = 1$ .

8. (10%) Find the volume of the solid formed by revolving the region bounded by the graph of  $y = x^2 + 1$ ,  $y = 0$ ,  $x = 0$ , and  $x = 1$  about the  $y$ -axis.

寒假愉快! 下學期見!