

微積分四系共同教學考題

九十一學年度微積分上學期第一次期中考

- 前四題爲每題十五分 後四題每題十分
 - 本次考試計算極限值時不可使用羅必達法則
 - 每題作答須有計算或推導過程 否則以零分計
 - 答案卷務必寫上姓名學號科系 否則以零分計
 - 不可使用含有計算功能之電子儀器設備 否則以零分計
1. Find a, b, c and d such that the cubic $f(x) = ax^3 + bx^2 + cx + d$ satisfies the indicated conditions.
 - (a) Relative maximum : $(3, 3)$
 - (b) Relative minimum : $(5, 1)$
 - (c) Inflection point : $(4, 2)$
 2. A rectangular package to be sent by a postal service can have a maximum combined length and girth (perimeter of a cross section) of 108 inches. Find the dimensions of the package of maximum volume the can be sent. (Assume the cross section is square.)

3. Let

$$f(x) = \begin{cases} x \sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

Show that f is continuous, but not differentiable, at $x = 0$. Show that g is differentiable at 0, and find $g'(0)$.

4. Analyze and sketch the graph of $f(x) = \frac{2(x^2-9)}{x^2-4}$.

5. Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.

6. Determine the slope of the graph of

$$3(x^2 + y^2)^2 = 100xy$$

at the point $(3, 1)$.

7. Prove that $|\cos x - \cos y| \leq |x - y|$ for all x and y .

8. Find the relative extrema of $f(x) = (x^2 - 4)^{2/3}$.