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11. $f(x)=\dfrac{1}{x+2}\quad\Rightarrow$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\dfrac{1}{x+h+2}-\dfrac{1}{x+2}}{h}=\lim_{h\to 0}\frac{\dfrac{x+2-(x+h+2)}{(x+2)(x+h+2)}}{h}$$

$$=\lim_{h\to 0}\frac{-h}{h(x+2)(x+h+2)}=-\lim_{h\to 0}\frac{1}{(x+2)(x+h+2)}=-\frac{1}{(x+2)^2}\text{ with domain }(-\infty,-2)\cup(-2,\infty).$$

12. $f(x)=-\dfrac{2}{\sqrt{x}}\quad\Rightarrow$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{-\dfrac{2}{\sqrt{x+h}}-\left(-\dfrac{2}{\sqrt{x}}\right)}{h}=2\lim_{h\to 0}\frac{\dfrac{-\sqrt{x}+\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$

$$=2\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=2\lim_{h\to 0}\frac{(x+h)-x}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=2\lim_{h\to 0}\frac{h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x+h}+\sqrt{x}\right)}=2\lim_{h\to 0}\frac{1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{2}{\sqrt{x}\sqrt{x}\left(2\sqrt{x}\right)}$$

$$=\frac{1}{x\sqrt{x}}\text{ with domain }(0,\infty).$$

16. $f(x)=3x^2-4x+2\quad\Rightarrow$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\left[3(x+h)^2-4(x+h)+2\right]-\left(3x^2-4x+2\right)}{h}$$

$$=\lim_{h\to 0}\frac{\left(3x^2+6xh+3h^2-4x-4h+2\right)-\left(3x^2-4x+2\right)}{h}=\lim_{h\to 0}\frac{h(6x+3h-4)}{h}$$

$$=\lim_{h\to 0}(6x+3h-4)=6x-4$$

The slope of the tangent line at $(2,6)$ is $f'(2)=6(2)-4=8$. An equation of the tangent line is $y-6=8(x-2)$ or $y=8x-10$.

26. $y=f(x)=x^2-\dfrac{1}{x}\quad\Rightarrow$

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\left[(x+h)^2-\dfrac{1}{x+h}\right]-\left(x^2-\dfrac{1}{x}\right)}{h}$$

$$=\lim_{h\to 0}\frac{\left(x^2+2xh+h^2-\dfrac{1}{x+h}\right)-\left(x^2-\dfrac{1}{x}\right)}{h}=\lim_{h\to 0}\frac{2xh+h^2-\dfrac{1}{x+h}+\dfrac{1}{x}}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2-\dfrac{x-(x+h)}{x(x+h)}}{h}=\lim_{h\to 0}\frac{h\left[2x+h+\dfrac{1}{x(x+h)}\right]}{h}=\lim_{h\to 0}\left[(2x+h)+\frac{1}{x(x+h)}\right]=2x+\frac{1}{x^2}$$

$\left.\dfrac{dy}{dx}\right|_{x=-1}=2(-1)+\dfrac{1}{(-1)^2}=-1$, so y is decreasing at the rate of 1 unit per unit change in x.

50. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x+1) = 1$, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(x^2 + 1\right) = 1$. Therefore, $\lim_{x \to 0} f(x) = 1$. Also, $f(0) = 0 + 1 = 1$, and so $\lim_{x \to 0} f(x) = f(0)$. Therefore, f is continuous at 0.

To show that f is not differentiable at 0, let $h < 0$ and consider $\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{(h+1) - 1}{h} = \lim_{h \to 0^-} 1 = 1$. Next, if $h > 0$, then $\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{\left[(0+h)^2 + 1\right] - 1}{h} = \lim_{h \to 0^+} h = 0$. This shows that $\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$ does not exist, and so by definition, f is not differentiable at 0.

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22. $f'(x) = \frac{d}{dx}\left(5x^{4/3} - \frac{2}{3}x^{3/2} + x^2 - 3x + 1\right) = \frac{20}{3}x^{1/3} - x^{1/2} + 2x - 3$

31. $y' = \frac{d}{dx}\left(x^{1/3} + x^{-1/2}\right) = \frac{1}{3}x^{-2/3} - \frac{1}{2}x^{-3/2} = \frac{1}{3x^{2/3}} - \frac{1}{2x^{3/2}}$

42. $F'(s) = \frac{d}{ds}\left(2 + \frac{1}{s}\right) = -\frac{1}{s^2} = -\frac{1}{9} \Rightarrow s^2 = 9 \Rightarrow s = \pm 3$. $F(3) = \frac{2(3)+1}{3} = \frac{7}{3}$ and $F(-3) = \frac{2(-3)+1}{-3} = \frac{5}{3}$, so the points are $\left(3, \frac{7}{3}\right)$ and $\left(-3, \frac{5}{3}\right)$.

58. We use the definition of the derivative to write

$$\lim_{h\to 0} \frac{\frac{1}{x+h} + \sqrt{x+h} - \frac{1}{x} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} + \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{d}{dx}\left(\sqrt{x}\right) = -\frac{1}{x^2} + \frac{1}{2\sqrt{x}}.$$

72. In order for f to be continuous at a, $\lim_{x\to a^-} f(x) = \lim_{x\to a^-} x^2 = a^2$ must be equal to $\lim_{x\to a^+} f(x) = \lim_{x\to a^+} (Ax + B) = Aa + B$, that is, $aA + B = a^2$. In order for f to be differentiable at a, we must have $\lim_{h\to 0^-} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0^+} \frac{f(a+h) - f(a)}{h}$ or $2a = A$. Therefore, $A = 2a$ and $B = a^2 - 2a^2 = -a^2$.