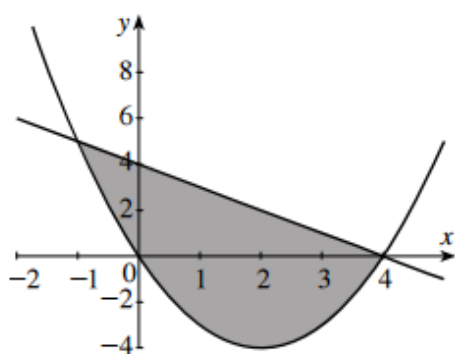


5.1

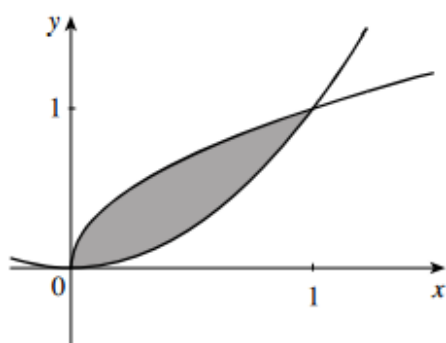
12. Solve $x^2 - 4x = -x + 4 \Leftrightarrow x^2 - 3x - 4 = (x - 4)(x + 1) = 0$, giving $(-1, 5)$ and $(4, 0)$ as the points of intersection. Thus,

$$\begin{aligned} A &= \int_{-1}^4 [(-x + 4) - (x^2 - 4x)] dx = \int_{-1}^4 (-x^2 + 3x + 4) dx \\ &= -\frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x \Big|_{-1}^4 = \frac{125}{6} \end{aligned}$$

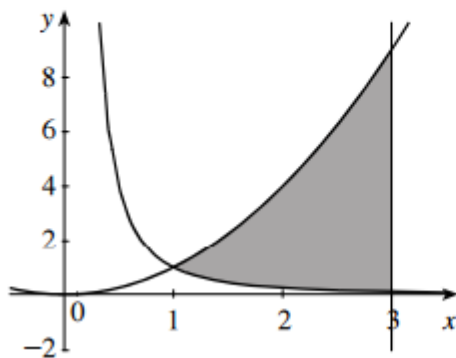


17. Solve $\sqrt{x} = x^2 \Leftrightarrow x = x^4 \Leftrightarrow x(x^3 - 1) = 0$, giving $(0, 0)$ and $(1, 1)$ as the points of intersection. Thus,

$$A = \int_0^1 (\sqrt{x} - x^2) dx = \frac{2}{3}x^{3/2} - \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{3}.$$

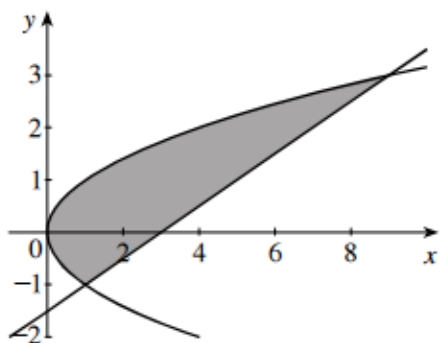


$$21. A = \int_1^3 \left(x^2 - \frac{1}{x^2} \right) dx = \frac{1}{3}x^3 + \frac{1}{x} \Big|_1^3 = \left(9 + \frac{1}{3} \right) - \left(\frac{1}{3} + 1 \right) = 8$$



28. Solve $y^2 = 2y + 3 \Leftrightarrow y^2 - 2y - 3 = (y - 3)(y + 1) = 0$, giving $(1, -1)$ and $(9, 3)$ as the points of intersection. Thus,

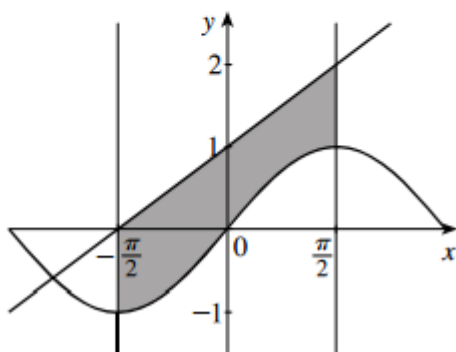
$$\begin{aligned} A &= \int_{-1}^3 \left[(2y + 3) - y^2 \right] dy = \int_{-1}^3 \left(-y^2 + 2y + 3 \right) dy \\ &= -\frac{1}{3}y^3 + y^2 + 3y \Big|_{-1}^3 = \frac{32}{3} \end{aligned}$$



$$32. A = \int_{-\pi/2}^{\pi/2} \left[\left(\frac{2}{\pi}x + 1 \right) - \sin x \right] dx$$

$$= 2 \int_0^{\pi/2} 1 dx = \pi$$

because $f(x) = \frac{2}{\pi}x - \sin x$ is odd.



5.2

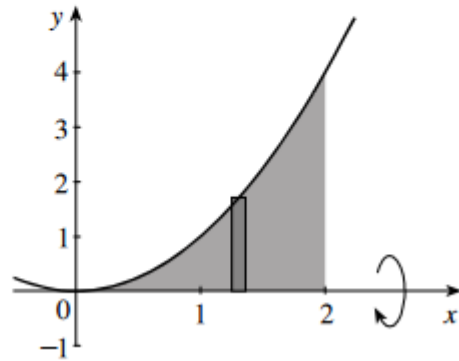
$$7. V = \pi \int_0^1 \left[\left(y^{1/3} \right)^2 - \left(y^{1/2} \right)^2 \right] dy = \pi \int_0^1 \left(y^{2/3} - y \right) dy$$

$$= \pi \left(\frac{3}{5} y^{5/3} - \frac{1}{2} y^2 \right) \Big|_0^1 = \frac{\pi}{10}$$

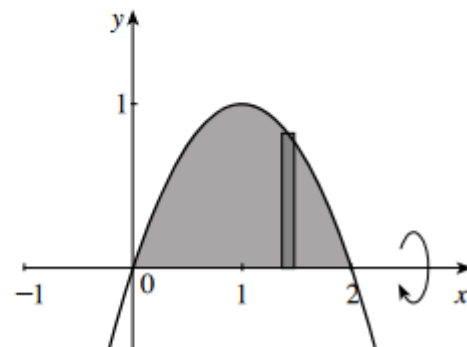
$$11. V = \pi \int_0^1 \left[\left(\frac{3}{2} - x^3 \right)^2 - \left(\frac{3}{2} - x \right)^2 \right] dx = \pi \int_0^1 \left(x^6 - 3x^3 - x^2 + 3x \right) dx$$

$$= \pi \left(\frac{1}{7} x^7 - \frac{3}{4} x^4 - \frac{1}{3} x^3 + \frac{3}{2} x^2 \right) \Big|_0^1 = \frac{47\pi}{84}$$

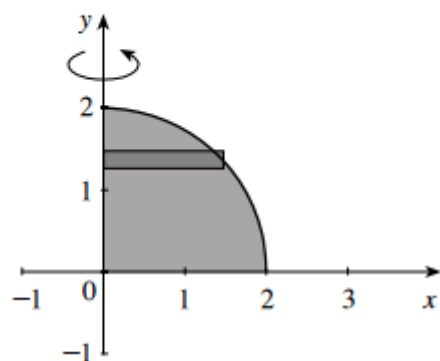
$$\begin{aligned}
 13. \quad V &= \pi \int_0^2 y^2 dx = \pi \int_0^2 (x^2)^2 dx = \pi \int_0^2 x^4 dx \\
 &= \pi \left(\frac{1}{5} x^5 \right) \Big|_0^2 = \frac{32\pi}{5}
 \end{aligned}$$



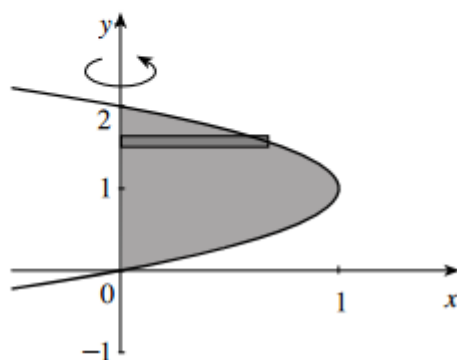
$$\begin{aligned}
 15. \quad V &= \pi \int_0^2 y^2 dx = \pi \int_0^2 (-x^2 + 2x)^2 dx \\
 &= \pi \int_0^2 (x^4 - 4x^3 + 4x^2) dx \\
 &= \pi \left(\frac{1}{5} x^5 - x^4 + \frac{4}{3} x^3 \right) \Big|_0^2 = \frac{16\pi}{15}
 \end{aligned}$$



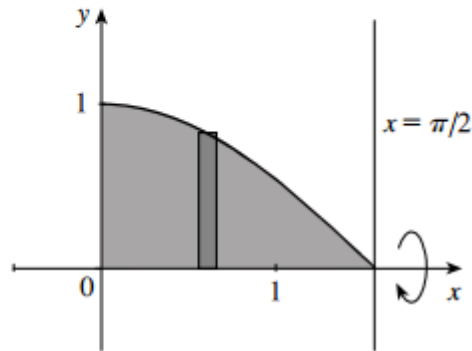
$$\begin{aligned}
 19. \quad V &= \pi \int_0^2 x^2 dy = \pi \int_0^2 (\sqrt{4-y^2})^2 dy \\
 &= \pi \int_0^2 (4-y^2) dy = \pi \left(4y - \frac{1}{3}y^3\right) \Big|_0^2 = \frac{16\pi}{3}
 \end{aligned}$$



$$\begin{aligned}
 20. \quad V &= \pi \int_0^2 x^2 dy = \pi \int_0^2 (-y^2 + 2y)^2 dy \\
 &= \pi \int_0^2 (y^4 - 4y^3 + 4y^2) dy \\
 &= \pi \left(\frac{1}{5}y^5 - y^4 + \frac{4}{3}y^3\right) \Big|_0^2 = \frac{16\pi}{15}
 \end{aligned}$$



$$\begin{aligned}
 25. \quad V &= \pi \int_0^{\pi/2} y^2 dx = \pi \int_0^{\pi/2} \cos^2 x dx \\
 &= \frac{\pi}{2} \int_0^{\pi/2} (1 + \cos 2x) dx \\
 &= \frac{\pi}{2} \left(x + \frac{1}{2} \sin 2x \right) \Big|_0^{\pi/2} = \frac{\pi^2}{4}
 \end{aligned}$$

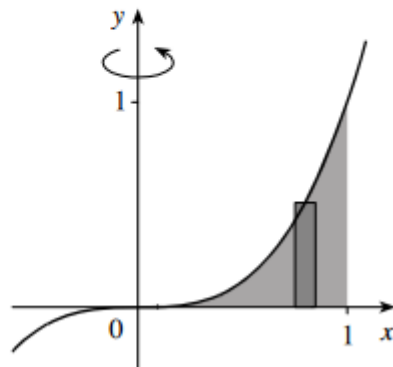


5.3

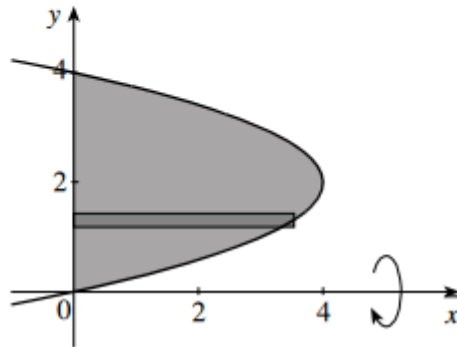
$$\begin{aligned}
 4. \quad V &= 2\pi \int_0^2 y(4 - y^2) dy = 2\pi \int_0^2 (4y - y^3) dy \\
 &= 2\pi \left(2y^2 - \frac{1}{4}y^4 \right) \Big|_0^2 = 8\pi
 \end{aligned}$$

$$\begin{aligned}
 6. \quad V &= 2\pi \int_0^2 (3 - x) \left[\left(\frac{1}{2}x^2 + 2 \right) - x^2 \right] dx = 2\pi \int_0^2 \left(\frac{1}{2}x^3 - \frac{3}{2}x^2 - 2x + 6 \right) dx \\
 &= 2\pi \left(\frac{1}{8}x^4 - \frac{1}{2}x^3 - x^2 + 6x \right) \Big|_0^2 = 12\pi
 \end{aligned}$$

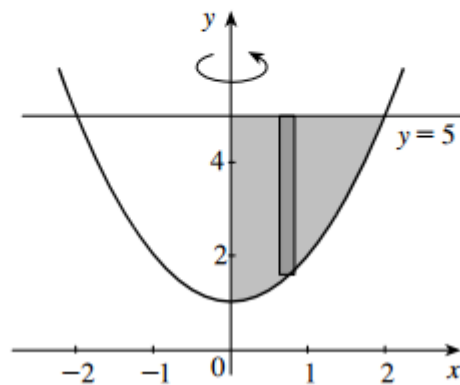
$$\begin{aligned}
 8. \quad V &= 2\pi \int_0^1 xy dx = 2\pi \int_0^1 x(x^3) dx \\
 &= 2\pi \int_0^1 x^4 dx = 2\pi \left(\frac{1}{5}x^5 \right) \Big|_0^1 = \frac{2\pi}{5}
 \end{aligned}$$



$$\begin{aligned}
 12. \quad V &= 2\pi \int_0^4 yx \, dy = 2\pi \int_0^4 y(-y^2 + 4y) \, dy \\
 &= 2\pi \int_0^4 (-y^3 + 4y^2) \, dy = 2\pi \left(-\frac{1}{4}y^4 + \frac{4}{3}y^3\right) \Big|_0^4 \\
 &= \frac{128\pi}{3}
 \end{aligned}$$



$$\begin{aligned}
 15. \quad V &= 2\pi \int_0^2 x \left[5 - (x^2 + 1)\right] \, dx \\
 &= 2\pi \int_0^2 (-x^3 + 4x) \, dx = 2\pi \left(-\frac{1}{4}x^4 + 2x^2\right) \Big|_0^2 = 8\pi
 \end{aligned}$$



16. To find the points of intersection of the two graphs, we solve $\frac{1}{2}x^2 = x \Leftrightarrow x(x - 2) = 0$, giving the points $(0, 0)$ and $(2, 2)$.

$$\begin{aligned} V &= 2\pi \int_0^2 x \left(x - \frac{1}{2}x^2 \right) dx = 2\pi \int_0^2 \left(x^2 - \frac{1}{2}x^3 \right) dx \\ &= 2\pi \left(\frac{1}{3}x^3 - \frac{1}{8}x^4 \right) \Big|_0^2 = \frac{4\pi}{3} \end{aligned}$$

