

## 4.1 Indefinite Integrals

$$2. \int (6x^2 - 2x + 1) dx = 2x^3 - x^2 + x + C$$

$$6. \int (2x^{2/3} - 4x^{1/3} + 4) dx = \frac{6}{5}x^{5/3} - 3x^{4/3} + 4x + C$$

$$8. \int x^{2/3} (x - 1) dx = \int (x^{5/3} - x^{2/3}) dx = \frac{3}{8}x^{8/3} - \frac{3}{5}x^{5/3} + C$$

$$12. \int \frac{x^2 + 1}{x^2} dx = \int (1 + x^{-2}) dx = x - x^{-1} + C = x - \frac{1}{x} + C$$

$$14. \int \frac{t^2 - 2\sqrt{t} + 1}{t^2} dt = \int (1 - 2t^{-3/2} + t^{-2}) dt = t + 4t^{-1/2} - t^{-1} + C = t + \frac{4}{\sqrt{t}} - \frac{1}{t} + C$$

$$19. \int (3 \sin x - 4 \cos x) dx = -3 \cos x - 4 \sin x + C$$

$$22. \int \sec u (\tan u + \sec u) du = \int (\sec u \tan u + \sec^2 u) du = \sec u + \tan u + C$$

$$24. \int \frac{\sin 2x}{\cos x} dx = \int \frac{2 \sin x \cos x}{\cos x} dx = \int 2 \sin x dx = -2 \cos x + C$$

## 4.2 Integration by Substitution

8. For  $I = \int x^2 (2x^3 - 1)^4 dx$ , let  $u = 2x^3 - 1 \Rightarrow du = 6x^2 dx \Rightarrow x^2 dx = \frac{1}{6} du$ . Then

$$I = \frac{1}{6} \int u^4 du = \frac{1}{30} u^5 + C = \frac{1}{30} (2x^3 - 1)^5 + C.$$

12. For  $I = \int x^2 (2x^3 - 1)^{-4} dx$ , let  $u = 2x^3 - 1 \Rightarrow du = 6x^2 dx \Rightarrow x^2 dx = \frac{1}{6} du$ . Then

$$I = \frac{1}{6} \int u^{-4} du = \frac{1}{6} \left( -\frac{1}{3} u^{-3} \right) + C = -\frac{1}{18} (2x^3 - 1)^{-3} + C.$$

15. For  $I = \int \sqrt{1 - 2x} dx$ , let  $u = 1 - 2x \Rightarrow du = -2 dx \Rightarrow dx = -\frac{1}{2} du$ . Then

$$\int \sqrt{1 - 2x} dx = -\frac{1}{2} \int \sqrt{u} du = -\frac{1}{2} \left( \frac{2}{3} u^{3/2} \right) + C = -\frac{1}{3} \sqrt{(1 - 2x)^3} + C.$$

21. For  $I = \int \frac{4u}{\sqrt{4-u^2}} du$ , let  $v = 4 - u^2 \Rightarrow dv = -2u du \Rightarrow u du = -\frac{1}{2} dv$ . Then

$$I = 4 \left( -\frac{1}{2} \right) \int \frac{dv}{v^{1/2}} = -2 \int v^{-1/2} dv = -2 \left( 2v^{1/2} \right) + C = -4\sqrt{4-u^2} + C.$$

23. For  $I = \int \frac{x}{\sqrt{x-1}} dx$ , let  $u = x - 1 \Rightarrow du = dx$  and  $x = u + 1$ . Then

$$I = \int \frac{(u+1)du}{u^{1/2}} = \int \left( u^{1/2} + u^{-1/2} \right) du = \frac{2}{3}u^{3/2} + 2u^{1/2} + C = \frac{2}{3}u^{1/2}(u+3) + C = \frac{2}{3}(x+2)\sqrt{x-1} + C.$$

28. For  $I = \int x \sin x^2 dx$ , let  $u = x^2 \Rightarrow du = 2x dx \Rightarrow x dx = \frac{1}{2} du$ . Then

$$I = \frac{1}{2} \int \sin u du = -\frac{1}{2} \cos u + C = -\frac{1}{2} \cos x^2 + C.$$

34. For  $I = \int \sqrt{\sin \theta} \cos \theta d\theta$ , let  $u = \sin \theta \Rightarrow du = \cos \theta d\theta$ . Then  $I = \int \sqrt{u} du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\sqrt{\sin^3 \theta} + C$ .

35. For  $I = \int \frac{\sin u^{-1}}{u^2} du$ , let  $v = u^{-1} \Rightarrow dv = -u^{-2} du$ . Then  $I = -\int \sin v dv = \cos v + C = \cos u^{-1} + C$ .

36. For  $I = \int \frac{\sin x}{(1 + \cos x)^3} dx$ , let  $u = 1 + \cos x \Rightarrow du = -\sin x dx$ . Then

$$I = -\int \frac{du}{u^3} = -\int u^{-3} du = \frac{1}{2}u^{-2} + C = \frac{1}{2(1 + \cos x)^2} + C.$$

38. For  $I = \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$ , let  $u = \sqrt{x} = x^{1/2} \Rightarrow du = \frac{1}{2}x^{-1/2} dx = \frac{1}{2\sqrt{x}} dx \Rightarrow \frac{1}{\sqrt{x}} dx = 2 du$ . Then

$$I = 2 \int \sin u du = -2 \cos u + C = -2 \cos \sqrt{x} + C.$$

41. For  $I = \int \frac{\sec x \tan x}{(\sec x - 1)^2} dx$ , let  $u = \sec x - 1 \Rightarrow du = \sec x \tan x dx$ . Then

$$I = \int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\sec x - 1} + C = \frac{1}{1 - \sec x} + C.$$

44. For  $I = \int \frac{1 + \sin x}{\cos^2 x} dx = \int \left( \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \right) dx = \int \sec^2 x dx + \int \frac{\sin x}{\cos^2 x} dx$ , let  $u = \cos x$  in the second integral.

$$\text{Then } du = -\sin x dx, \text{ so } I = \int \sec^2 x dx - \int \frac{du}{u^2} = \tan x + \frac{1}{u} + C = \tan x + \frac{1}{\cos x} + C = \tan x + \sec x + C.$$

45. For  $I = \int x \sqrt{x-4} dx$ , let  $u = x - 4 \Rightarrow du = dx$  and  $x = u + 4$ . Then

$$\begin{aligned} I &= \int (u+4) \sqrt{u} du = \int (u^{3/2} + 4u^{1/2}) dx = \frac{2}{5}u^{5/2} + \frac{8}{3}u^{3/2} + C \\ &= \frac{2}{15}u^{3/2}(3u+20) + C = \frac{2}{15}(3x+8)\sqrt{(x-4)^3} + C. \end{aligned}$$

47. For  $I = \int x^3 (x^2 + 1)^{5/2} dx$ , let  $u = x^2 + 1 \Rightarrow du = 2x dx$  and  $x^2 = u - 1$ . Then

$$\begin{aligned} I &= \int x^2 (x^2 + 1)^{5/2} (x dx) = \frac{1}{2} \int (u-1) u^{5/2} du = \frac{1}{2} \int (u^{7/2} - u^{5/2}) du = \frac{1}{2} \left( \frac{2}{9}u^{9/2} - \frac{2}{7}u^{7/2} \right) + C \\ &= \frac{1}{9} (x^2 + 1)^{9/2} - \frac{1}{7} (x^2 + 1)^{7/2} + C. \end{aligned}$$