

$$24. \lim_{x \rightarrow a} \frac{f(x) + g(x)}{2h(x)} = \frac{\lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)}{2 \lim_{x \rightarrow a} h(x)} = \frac{2 + 4}{2(-1)} = -3$$

$$68. \lim_{x \rightarrow 0} \frac{\tan 2x}{3x} = \lim_{x \rightarrow 0} \left( \frac{\sin 2x}{\cos 2x} \cdot \frac{1}{3x} \right) = \left( \lim_{x \rightarrow 0} \frac{1}{\cos 2x} \right) \left[ \lim_{x \rightarrow 0} \frac{\sin 2x}{\frac{2}{3}(2x)} \right] = 1 \cdot \frac{2}{3} \cdot \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = \frac{2}{3} \cdot 1 = \frac{2}{3}$$

91. Let  $g(x) = -x^2$  and  $h(x) = x^2$  for all real  $x$ . Then  $g(x) \leq f(x) \leq h(x)$  for all  $x$ . Since  $\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} h(x) = 0$ , the result follows from the Squeeze Theorem.

93. Let  $f(x) = \frac{1}{x}$  and  $g(x) = -\frac{1}{x}$ . Then  $\lim_{x \rightarrow 0} f(x)$  and  $\lim_{x \rightarrow 0} g(x)$  both fail to exist. But

$\lim_{x \rightarrow 0} [f(x) + g(x)] = \lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{x} \right) = \lim_{x \rightarrow 0} 0 = 0$ . This example does not contradict the Sum Law of Limits because the law is valid only if both  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  exist.

28. Since  $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2} = \lim_{x \rightarrow -2} \frac{(x - 2)(x + 2)}{x + 2} = \lim_{x \rightarrow -2} (x - 2) = -4$ , we define  $f(-2) = k = -4$ , that is, take  $k = -4$ .

29. We require that  $f(1) = 4 = \lim_{x \rightarrow 1^-} (ax + b) = \lim_{x \rightarrow 1^+} (2ax - b)$ , or  $a + b = 4$  and  $2a - b = 4$ . Solving the last two equations simultaneously, we obtain  $a = \frac{8}{3}$  and  $b = \frac{4}{3}$ .

60.  $f(x) = x^2 - 4x + 6$  is continuous on  $[0, 3]$ .  $f(0) = 6$  and  $f(3) = 3$ . Since  $f(3) \leq 3 \leq f(0)$ , there exists a number  $c$  in  $[0, 3]$  such that  $f(c) = 3$ . To find  $c$  we solve  $x^2 - 4x + 6 = 3 \Rightarrow x^2 - 4x + 3 = (x - 3)(x - 1) = 0$  giving  $x = 1$  or  $3$ . Therefore  $c = 1$  or  $3$ .

64.  $f(x) = x^4 - 2x^3 - 3x^2 + 7$  is continuous on  $[1, 2]$ .  $f(1) = 3 > 0$  and  $f(2) = -5 < 0$ . Therefore, by Theorem 7,  $f(x) = 0$  has at least one root in  $(1, 2)$ .