Calculus II

2022/06/14

Final Test

考試注意事項：

1. 請用 A4 空白紙，手寫作答。
2. 請在 e 學院上以 第 18 周作業 型式，用 PDF 檔 回傳答案卷。
3. 未依以上規定，不予計分。

試題：

1. (15%) Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$ does not exist.
2. (15%) Use **Lagrange multipliers** to find the maximum and minimum values of the function $f(x,y)=x^2-2y$ subject to $x^2+y^2=9$.
3. (15%) Evaluate $\displaystyle\int_0^1 \int_y^1 \frac{\sin x}{x}\, dxdy$.
4. (15%) Evaluate
$$\iint_R \cos\left(\frac{x-y}{x+y}\right) dA$$
where R is the trapezoidal region with vertices $(1,0)$, $(2,0)$, $(0,2)$, and $(0,1)$.
5. (10%) Let $w=x^2y+y^2z^3$, where $x=r\cos s$, $y=r\sin s$ and $z=re^s$. Use the method of the **chain rule** to find the value of $\partial w/\partial s$ when $r=1$ and $s=0$.
6. (10%) Find the directional derivative of $f(x,y)=e^x\cos 2y$ at the point $(0,\dfrac{\pi}{4})$ in the direction $\mathbf{v}=2\mathbf{i}+3\mathbf{j}$.
7. (10%) Find equations of the tangent plane and normal line to the graph of the function f defined by $f(x,y)=4x^2+y^2+2$ at the point where $x=1$ and $y=1$.
8. (10%) Find the relative extrema of $f(x,y)=x^3+y^2-2xy+7x-8y+2$.

祝 大 家 疫 情 平 安