Calculus I

2020/12/01

Second Midterm

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。
3. 未依題目要求作答，不予計分。

試題：

1. (15%) Find the **absolute extreme values** of the function $f(x) = 3x^4 - 4x^3 - 8$ on $[-1, 2]$.

2. (15%) Find the **relative extrema** of $f(x) = x^4 - 4x^3 + 12$ using the **First Derivative Test**.

3. (15%) Find the **horizontal asymptotes** of the graph of the function

$$f(x) = \frac{3x}{\sqrt{x^2+1}}.$$

4. (15%) If $y = \int_0^{x^3} \cos t^2 \, dt$, what is $\frac{dy}{dx}$?

5. (10%) Find the **relative extrema** of the function $f(x) = x^3 - 3x^2 - 24x + 32$ using the **Second Derivative Test**.

6. (10%) Find $\int (3x^5 - 2x^3 + 2 - 3x^{-1/3}) \, dx$.

7. (10%) Evaluate the limit after **finding the sum** (as a function of n) using the summation formulas:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left[1 + 2\left(\frac{k}{n}\right)^2\right]\left(\frac{2}{n}\right)$$

8. (10%) Evaluate $\int_0^2 x\sqrt{x^2+4} \, dx$.