

Final Test

考試注意事項：

1. 答案紙直行對折，兩直攔書寫作答。
2. 無清楚計算過程，不予計分。
3. 未依題目要求作答，不予計分。

試題：

1. (15%) Find the relative extrema of $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.
2. (15%) Find the absolute extreme values of $f(x, y) = 2x^2 + y^2 - 2y + 1$ subject to the constraint $x^2 + y^2 = 4$.

3. (15%) Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$.

4. (15%) Evaluate

$$\iint_R \cos\left(\frac{x-y}{x+y}\right) dA$$

where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$, and $(0, 1)$.

5. (10%) Let $w = 2x^2y$, where $x = u^2 + v^2$ and $y = u^2 - v^2$. Find $\partial w / \partial u$.
6. (10%) Find the direction derivative of $f(x, y) = 4 - 2x^2 - y^2$ at the point $(1, 1)$ in the direction of the unit vector \mathbf{u} that makes an angle of $\pi/3$ radians with positive x -axis.
7. (10%) Find the equations of the tangent plane to the ellipsoid with equation $4x^2 + y^2 + 4z^2 = 16$ at the point $(1, 2, \sqrt{2})$.

8. (10%) Evaluate the iterated integral: $\int_0^1 \int_1^2 3x^2 y dy dx$.