Calculus II

Second Midterm

考試注意事項：

1. 答案紙直行對折，兩直攔書寫作答。

2. 無清楚計算過程，不予計分。

3. 未依題目要求作答，不予計分。

試題：

1. (20%) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ is convergent or divergent.

2. (20%) Find a power series representation for $1/(1+x)^2$ on $(-1, 1)$ by **differentiating a power series** of $1/(1+x)$. Determine the **interval of convergence** of this power series representation for $1/(1+x)^2$.

3. (20%) Let $f(x) = e^x$. Find the Maclaurin series of f, and determine **its interval of convergence**.

4. (10%) Show that the alternating harmonic series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

converges.

5. (5%, 5%) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$ if $x = t^2 - 4$ and $y = t^3 - 3t$.

6. (10%) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$ does not exist.

7. (10%) Let $f(x, y, z) = xe^{yz}$. Compute f_{xzy}.