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18. $\lim_{x\to 0^+} \dfrac{1}{1+e^{1/x}} = 0$ since $e^{1/x} \to \infty$ as $x \to 0^+$.

24. $f'(x) = \dfrac{d}{dx}\left(x^2e^{-2x}\right) = x^2\dfrac{d}{dx}\left(e^{-2x}\right) + e^{-2x}\dfrac{d}{dx}\left(x^2\right) = -2x^2e^{-2x} + 2xe^{-2x} = 2xe^{-2x}(1-x)$

26. $h'(t) = \dfrac{d}{dt}\left(\dfrac{e^t - e^{-t}}{e^t + e^{-t}}\right) = \dfrac{d}{dt}\left(\dfrac{e^{2t}-1}{e^{2t}+1}\right) = \dfrac{\left(e^{2t}+1\right)\left(2e^{2t}\right) - \left(e^{2t}-1\right)\left(2e^{2t}\right)}{\left(e^{2t}+1\right)^2} = \dfrac{4e^{2t}}{\left(e^{2t}+1\right)^2}$

28. $g'(x) = \dfrac{d}{dx}\left(e^{-2x}\cos 3x\right) = e^{-2x}(-3\sin 3x) + (\cos 3x)\left(-2e^{-2x}\right) = -e^{-2x}(3\sin 3x + 2\cos 3x)$

30. $y' = \dfrac{d}{dx}\left(e^{-1/x}\right) = e^{-1/x}\left[-\left(-\dfrac{1}{x^2}\right)\right] = \dfrac{e^{-1/x}}{x^2}$

32. $h'(x) = \dfrac{d}{dx}\left(e^{2x} - e^{-3x}\right)^5 = 5\left(e^{2x} - e^{-3x}\right)^4\left(2e^{2x} + 3e^{-3x}\right)$

34. $g'(x) = \dfrac{d}{dx}\ln\left(e^x + e^{-x}\right) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

36. $h'(x) = \dfrac{d}{dx}\left[\tan\left(e^{2x} + \ln x\right)\right] = \left[\sec^2\left(e^{2x} + \ln x\right)\right]\left(2e^{2x} + \dfrac{1}{x}\right) = \dfrac{1}{x}\left(2xe^{2x} + 1\right)\sec^2\left(e^{2x} + \ln x\right)$

38. $y' = \dfrac{d}{dx}\left(e^{-x}\tan e^x\right) = -e^{-x}\tan e^x + \left(e^{-x}\sec^2 e^x\right)e^x = \dfrac{e^x\sec^2 e^x - \tan e^x}{e^x}$

42. $e^{xy} - x^2 + y^2 = 5 \Rightarrow e^{xy}\left(y + xy'\right) - 2x + 2yy' = 0 \Leftrightarrow y'\left(xe^{xy} + 2y\right) + ye^{xy} - 2x = 0 \Leftrightarrow y' = \dfrac{2x - ye^{xy}}{xe^{xy} + 2y}$

44. $x\ln y + e^{-x} - ye^y = 0 \Rightarrow \ln y + \dfrac{x}{y}y' - e^{-x} - \left(e^y + ye^y\right)y' = 0 \Leftrightarrow y'\left(\dfrac{x}{y} - e^y - ye^y\right) - e^{-x} + \ln y = 0 \Leftrightarrow$

$y' = \dfrac{y\left(e^{-x} - \ln y\right)}{x - ye^y - y^2e^y}$

49. $y = xe^{-x} \Rightarrow y' = e^{-x} - xe^{-x} = (1-x)e^{-x} \Rightarrow y'|_1 = 0$. Thus, the slope of the required tangent line is $m = 0$, and an equation of the line is $y - e^{-1} = 0(x-1)$ or $y = 1/e$.

50. $xe^y + 2x + y = 3 \Rightarrow e^y + xe^yy' + 2 + y' = 0$. Substituting $x = 1$ and $y = 0$ into this equation gives $1 + y' + 2 + y' = 0$ or $y'|_{(1,0)} = -\frac{3}{2}$, so the slope of the required tangent line is $m = -\frac{3}{2}$, and an equation is $y - 0 = -\frac{3}{2}(x-1)$ or $y = -\frac{3}{2}x + \frac{3}{2}$.

85. Let $u = -x^2$, so $\int xe^{-x^2}\,dx = -\frac{1}{2}e^{-x^2} + C$.

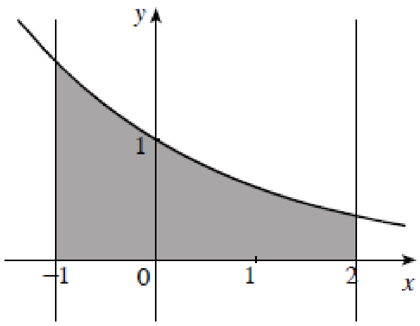
91. $I = \displaystyle\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$. Let $u = e^x - e^{-x}$, so $du = \left(e^x + e^{-x}\right)dx$. Then $I = \displaystyle\int \frac{du}{u} = \ln|u| + C = \ln\left|e^x - e^{-x}\right| + C$.

92. $I = \displaystyle\int \frac{e^{-1/x}}{x^2}\,dx$. Let $u = -\dfrac{1}{x}$, so $du = \dfrac{dx}{x^2}$. Then $I = \int e^u\,du = e^u + C = e^{-1/x} + C$.

94. $\int_{-1}^{0} \frac{1}{1+e^{-2x}}\,dx = \int_{-1}^{0} \frac{e^{2x}}{e^{2x}+1}\,dx = \frac{1}{2}\ln\left(e^{2x}+1\right)\Big|_{-1}^{0} = \frac{1}{2}\left[\ln 2 - \ln\left(e^{-2}+1\right)\right]$

101. Let $u = (\ln x)^2 \Rightarrow du = \frac{2\ln x}{x}\,dx \Rightarrow \frac{\ln x}{x}\,dx = \frac{1}{2}du$, $x = 1 \Rightarrow u = 0$, and $x = e \Rightarrow u = 1$. Then $\int_1^e \frac{\ln x}{x} e^{(\ln x)^2}\,dx = \frac{1}{2}\int_0^1 e^u\,du = \frac{1}{2}e^u\Big|_0^1 = \frac{1}{2}(e-1)$.

102. $A = \int_{-1}^{2} e^{-x/2}\,dx = -2e^{-x/2}\Big|_{-1}^{2}$
$= -2\left(e^{-1} - e^{1/2}\right) = 2\left(\sqrt{e} - 1/e\right)$



116. Differentiating the equation with respect to x, we have $\frac{d}{dx}\int_0^y e^t\,dt + \frac{d}{dx}\int_0^x \cos t\,dt = 0$. Using Part 1 of the Fundamental Theorem of Calculus and the Chain Rule, we have $e^y \cdot \frac{d}{dx}(y) + \cos x = 0 \Leftrightarrow e^y \frac{dy}{dx} = -\cos x \Leftrightarrow \frac{dy}{dx} = -e^{-y}\cos x$.

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22. $h(t) = 4^{t-1} \Rightarrow h'(t) = (\ln 4)\, 4^{t-1}$

24. $f(u) = 2^{u^2} \Rightarrow f'(u) = \ln 2\left(2^{u^2} \cdot 2u\right) = (2\ln 2)\, u \cdot 2^{u^2}$

27. $f(x) = x^e + e^x \Rightarrow f'(x) = ex^{e-1} + e^x$

39. $y = (x+2)^{1/x} \Rightarrow \ln y = \ln(x+2)^{1/x} = \frac{1}{x}\ln(x+2) \Rightarrow \frac{y'}{y} = -\frac{1}{x^2}\ln(x+2) + \frac{1}{x}\cdot\frac{1}{x+2} \Rightarrow$
$y' = \left[\frac{1}{x(x+2)} - \frac{\ln(x+2)}{x^2}\right](x+2)^{1/x}$

49. Let $u = 1 + 3^x$, so $du = 3^x \ln 3\,dx$. Then $\int \frac{3^x}{1+3^x}\,dx = \frac{1}{\ln 3}\int \frac{du}{u} = \frac{\ln u}{\ln 3} + C = \frac{\ln(3^x+1)}{\ln 3} + C$.

50. Let $u = \log x$, so $du = \frac{1}{\ln 10}\frac{dx}{x}$. Then $\int \frac{\sqrt{\log x}}{x}\,dx = \ln 10 \int u^{1/2}\,du = \frac{2\ln 10}{3}u^{3/2} + C = \frac{2\ln 10}{3}(\log x)^{3/2} + C$ or $\frac{2(\ln x)^{3/2}}{3\sqrt{\ln 10}} + C$.