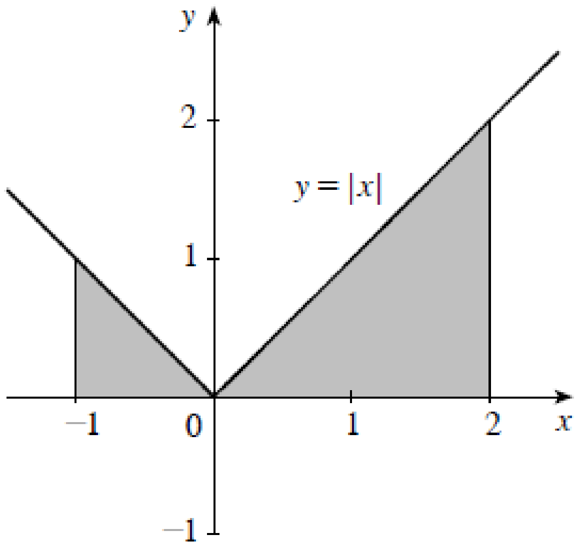
4.4

15. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2c_k}{c_k^2+1}\,\Delta x = \int_1^2 \frac{2x}{x^2+1}\,dx$

16. $\lim_{n\to\infty} \sum_{k=1}^{n} c_k \cos c_k \,\Delta x = \int_0^{\pi/2} x\cos x\,dx$

22. $\int_{-1}^{2} |x|\,dx = \frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) = \frac{5}{2}$

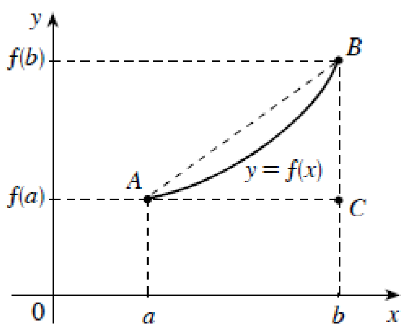


48. $-|f(x)| \le f(x) \le |f(x)|$ for all x in $[a,b]$, so by Property 5 of the definite integral,
$-\int_a^b |f(x)|\,dx = \int_a^b [-|f(x)|]\,dx \le \int_a^b f(x)\,dx \le \int_a^b |f(x)|\,dx \Rightarrow \left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx.$

50. Without loss of generality, we may assume that $f(x) > 0$ on $[a,b]$. Since f is concave upward on $[a,b]$, the graph of f lies below the chord connecting A and B. Therefore,

$$\left[\begin{array}{c}\text{area of}\\ \text{rectangle } abCA\end{array}\right] \le \left[\begin{array}{c}\text{area under}\\ \text{the graph of } f\end{array}\right] \le \left[\begin{array}{c}\text{area of}\\ \text{trapezoid } abBA\end{array}\right] \Rightarrow$$

$(b-a)\,f(a) \le \int_a^b f(x)\,dx \le \frac{1}{2}(b-a)\left[f(a)+f(b)\right].$



4.5

4. $G'(x) = \frac{d}{dx} \int_{-1}^{x} t\sqrt{t^2+1}\,dt = x\sqrt{x^2+1}$

6. $h'(x) = \frac{d}{dx} \int_{x}^{3} \frac{t}{\sqrt{t+1}}\,dt = -\frac{d}{dx} \int_{3}^{x} \frac{t}{\sqrt{t+1}}\,dt = -\frac{x}{\sqrt{x+1}}$

8. $G'(x) = \frac{d}{dx} \int_{0}^{x^2} t \sin t\,dt = \left(x^2 \sin x^2\right) \frac{d}{dx}\left(x^2\right) = 2x^3 \sin x^2$

11. $F'(x) = \frac{d}{dx} \int_{1}^{\cos x} \frac{t^2}{t+1}\,dt = \frac{\cos^2 x}{\cos + 1} \cdot \frac{d}{dx}(\cos x) = -\frac{\sin x \cos^2 x}{\cos x + 1}$

12. $G'(x) = \frac{d}{dx} \int_{\sqrt{x}}^{5} \frac{\sin t^2}{t}\,dt = -\frac{d}{dx} \int_{5}^{x^{1/2}} \frac{\sin t^2}{t}\,dt = -\frac{\sin x}{\sqrt{x}} \cdot \frac{d}{dx}\left(x^{1/2}\right) = -\frac{\sin x}{2x}$

20. $\int_1^2 \frac{3x^4 - 2x^2 + 1}{2x^2}\,dx = \frac{1}{2}\int_1^2 \left(3x^2 - 2 + x^{-2}\right) dx = \frac{1}{2}\left(x^3 - 2x - \frac{1}{x}\right)\Big|_1^2 = \frac{1}{2}\left[\left(8 - 4 - \frac{1}{2}\right) - (1 - 2 - 1)\right] = \frac{11}{4}$

21. $\int_4^9 \frac{x-1}{\sqrt{x}}\,dx = \int_4^9 \left(x^{1/2} - x^{-1/2}\right) dx = \frac{2}{3}x^{3/2} - 2x^{1/2}\Big|_4^9 = \frac{2}{3}\sqrt{x}\,(x-3)\Big|_4^9 = \frac{2}{3}\left[\sqrt{9}\,(9-3) - \sqrt{4}\,(4-3)\right] = \frac{32}{3}$

25. $\int_{\pi/6}^{\pi/4} \sec^2 t\,dt = \tan t\big|_{\pi/6}^{\pi/4} = \tan\frac{\pi}{4} - \tan\frac{\pi}{6} = 1 - \frac{\sqrt{3}}{3} = \frac{3-\sqrt{3}}{3}$

27. $\int_0^\pi \sin 2x \cos x\,dx = \int_0^\pi (2\sin x \cos x)\cos x\,dx = 2\int_0^\pi \cos^2 x \sin x\,dx = -\frac{2}{3}\cos^3 x\Big|_0^\pi = -\frac{2}{3}(-1-1) = \frac{4}{3}$

28. $\int_0^\pi |\cos x|\,dx = \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{\pi} \cos x\,dx = [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi} = (1-0) - (0-1) = 2$

35. Let $u = t^2 - 1$, so $du = 2t\,dt$, $t = 1 \Rightarrow u = 0$, and $t = 2 \Rightarrow u = 3$. Then
$\int_1^2 8t\left(t^2-1\right)^7 dt = 4\int_0^3 u^7\,du = \frac{1}{2}u^8\Big|_0^3 = \frac{1}{2}\left(3^8\right) = \frac{6561}{2}$.

37. Let $v = 5 - u$, so $dv = -du$, $u = 1 \Rightarrow v = 4$, and $u = 4 \Rightarrow v = 1$. Then
$\int_1^4 \sqrt[3]{5-u}\,du = -\int_4^1 v^{1/3}\,dv = \int_1^4 v^{1/3}\,dv = \frac{3}{4}v^{4/3}\Big|_1^4 = \frac{3}{4}\left(4\sqrt[3]{4} - 1\right)$.

39. Let $u=\sqrt{x}+1$, so $du=\frac{1}{2}x^{-1/2}\,dx=\dfrac{1}{2\sqrt{x}}\,dx\Rightarrow\dfrac{dx}{\sqrt{x}}=2\,du$, $x=1\Rightarrow u=2$, and $x=4\Rightarrow u=3$. Thus,
$$\int_1^4\frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}\,dx=2\int_2^3\frac{du}{u^2}=2\int_2^3u^{-2}\,du=-\frac{2}{u}\bigg|_2^3=-\tfrac{2}{3}+1=\tfrac{1}{3}.$$

42. Let $u=\cos\theta$, so $du=-\sin\theta d\theta\Rightarrow\sin\theta\,d\theta=-du$, $\theta=0\Rightarrow u=1$, and $\theta=\frac{\pi}{2}\Rightarrow u=0$. Then
$$\int_0^{\pi/2}\sqrt{\cos\theta}\sin\theta\,d\theta=-\int_1^0u^{1/2}\,du=\int_0^1u^{1/2}\,du=\tfrac{2}{3}u^{3/2}\Big|_0^1=\tfrac{2}{3}.$$

60. $$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^{1/3}=\int_0^1x^{1/3}\,dx=\tfrac{3}{4}x^{4/3}\Big|_0^1=\tfrac{3}{4}$$

62. $$\lim_{n\to\infty}\frac{\pi}{2n}\sum_{k=1}^{n}\cos\left(\frac{k\pi}{2n}\right)=\int_0^{\pi/2}\cos x\,dx=\sin x|_0^{\pi/2}=\sin\tfrac{\pi}{2}-\sin0=1$$

88. $$\frac{d}{dy}\left[\int_0^x\sqrt{3+2\cos t}\,dt+\int_0^y\sin t\,dt\right]=\frac{d}{dy}(0)=0\Rightarrow\sqrt{3+2\cos x}\frac{dx}{dy}+\sin y=0\Rightarrow\frac{dx}{dy}=-\frac{\sin y}{\sqrt{3+2\cos x}}$$

92. Let $F(x)=\int_2^x\sqrt{5+t^2}\,dt$. Then
$$F'(2)=\lim_{h\to0}\frac{F(2+h)-F(2)}{h}=\lim_{h\to0}\frac{\int_0^{2+h}\sqrt{5+t^2}\,dt-\int_0^2\sqrt{5+t^2}\,dt}{h}=\lim_{h\to0}\frac{\int_2^{2+h}\sqrt{5+t^2}\,dt}{h}.$$ Next, using FTC1, we find $F'(x)=\frac{d}{dx}\int_2^x\sqrt{5+t^2}\,dt=\sqrt{5+x^2}\Rightarrow F'(2)=\sqrt{5+2^2}=3$, so we have $\lim_{h\to0}\dfrac{1}{h}\int_2^{2+h}\sqrt{5+t^2}\,dt=3$.