4.1

2. $\int \left(6x^2 - 2x + 1\right) dx = 2x^3 - x^2 + x + C$

6. $\int \left(2x^{2/3} - 4x^{1/3} + 4\right) dx = \frac{6}{5}x^{5/3} - 3x^{4/3} + 4x + C$

8. $\int x^{2/3}(x-1)\,dx = \int \left(x^{5/3} - x^{2/3}\right) dx = \frac{3}{8}x^{8/3} - \frac{3}{5}x^{5/3} + C$

12. $\displaystyle\int \frac{x^2+1}{x^2}\,dx = \int \left(1 + x^{-2}\right) dx = x - x^{-1} + C = x - \frac{1}{x} + C$

14. $\displaystyle\int \frac{t^2 - 2\sqrt{t} + 1}{t^2}\,dt = \int \left(1 - 2t^{-3/2} + t^{-2}\right) dt = t + 4t^{-1/2} - t^{-1} + C = t + \frac{4}{\sqrt{t}} - \frac{1}{t} + C$

19. $\int (3\sin x - 4\cos x)\,dx = -3\cos x - 4\sin x + C$

22. $\int \sec u\,(\tan u + \sec u)\,du = \int \left(\sec u \tan u + \sec^2 u\right) du = \sec u + \tan u + C$

24. $\displaystyle\int \frac{\sin 2x}{\cos x} dx = \int \frac{2\sin x \cos x}{\cos x}\,dx = \int 2\sin x\,dx = -2\cos x + C$

4.2

8. For $I = \int x^2\left(2x^3 - 1\right)^4 dx$, let $u = 2x^3 - 1 \Rightarrow du = 6x^2\,dx \Rightarrow x^2\,dx = \frac{1}{6}\,du$. Then
$I = \frac{1}{6}\int u^4\,du = \frac{1}{30}u^5 + C = \frac{1}{30}\left(2x^3 - 1\right)^5 + C.$

12. For $I = \int x^2\left(2x^3 - 1\right)^{-4} dx$, let $u = 2x^3 - 1 \Rightarrow du = 6x^2\,dx \Rightarrow x^2\,dx = \frac{1}{6}\,du$. Then
$I = \frac{1}{6}\int u^{-4}\,du = \frac{1}{6}\left(-\frac{1}{3}u^{-3}\right) + C = -\frac{1}{18}\left(2x^3 - 1\right)^{-3} + C.$

15. For $I = \int \sqrt{1-2x}\,dx$, let $u = 1 - 2x \Rightarrow du = -2\,dx \Rightarrow dx = -\frac{1}{2}\,du$. Then
$\int \sqrt{1-2x}\,dx = -\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C = -\frac{1}{3}\sqrt{(1-2x)^3} + C.$

21. For $I = \int \frac{4u}{\sqrt{4-u^2}}\,du$, let $v = 4 - u^2 \Rightarrow dv = -2u\,du \Rightarrow u\,du = -\frac{1}{2}\,dv$. Then

$$I = 4\left(-\frac{1}{2}\right)\int \frac{dv}{v^{1/2}} = -2\int v^{-1/2}\,dv = -2\left(2v^{1/2}\right) + C = -4\sqrt{4-u^2} + C.$$

23. For $I = \int \frac{x}{\sqrt{x-1}}\,dx$, let $u = x - 1 \Rightarrow du = dx$ and $x = u + 1$. Then

$$I = \int \frac{(u+1)\,du}{u^{1/2}} = \int \left(u^{1/2} + u^{-1/2}\right)du = \tfrac{2}{3}u^{3/2} + 2u^{1/2} + C = \tfrac{2}{3}u^{1/2}(u+3) + C = \tfrac{2}{3}(x+2)\sqrt{x-1} + C.$$

28. For $I = \int x \sin x^2\,dx$, let $u = x^2 \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}\,du$. Then

$$I = \tfrac{1}{2}\int \sin u\,du = -\tfrac{1}{2}\cos u + C = -\tfrac{1}{2}\cos x^2 + C.$$

34. For $I = \int \sqrt{\sin\theta}\cos\theta\,d\theta$, let $u = \sin\theta \Rightarrow du = \cos\theta\,d\theta$. Then $I = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\sqrt{\sin^3\theta} + C$.

35. For $I = \int \frac{\sin u^{-1}}{u^2}\,du$, let $v = u^{-1} \Rightarrow dv = -u^{-2}\,du$. Then $I = -\int \sin v\,dv = \cos v + C = \cos u^{-1} + C$.

36. For $I = \int \frac{\sin x}{(1+\cos x)^3}\,dx$, let $u = 1 + \cos x \Rightarrow du = -\sin x\,dx$. Then

$$I = -\int \frac{du}{u^3} = -\int u^{-3}\,du = \tfrac{1}{2}u^{-2} + C = \frac{1}{2(1+\cos x)^2} + C.$$

38. For $I = \int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$, let $u = \sqrt{x} = x^{1/2} \Rightarrow du = \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2\sqrt{x}}\,dx \Rightarrow \frac{1}{\sqrt{x}}\,dx = 2\,du$. Then

$$I = 2\int \sin u\,du = -2\cos u + C = -2\cos\sqrt{x} + C.$$

41. For $I = \int \frac{\sec x \tan x}{(\sec x - 1)^2}\,dx$, let $u = \sec x - 1 \Rightarrow du = \sec x \tan x\,dx$. Then

$$I = \int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\sec x - 1} + C = \frac{1}{1 - \sec x} + C.$$

44. For $I = \int \frac{1+\sin x}{\cos^2 x}\,dx = \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right)dx = \int \sec^2 x\,dx + \int \frac{\sin x}{\cos^2 x}\,dx$, let $u = \cos x$ in the second integral.

Then $du = -\sin x\,dx$, so $I = \int \sec^2 x\,dx - \int \frac{du}{u^2} = \tan x + \frac{1}{u} + C = \tan x + \frac{1}{\cos x} + C = \tan x + \sec x + C$.

45. For $I = \int x\sqrt{x-4}\,dx$, let $u = x - 4 \Rightarrow du = dx$ and $x = u + 4$. Then

$$I = \int (u+4)\sqrt{u}\,du = \int \left(u^{3/2} + 4u^{1/2}\right) dx = \tfrac{2}{5}u^{5/2} + \tfrac{8}{3}u^{3/2} + C$$

$$= \tfrac{2}{15}u^{3/2}(3u+20) + C = \tfrac{2}{15}(3x+8)\sqrt{(x-4)^3} + C.$$

47. For $I = \int x^3\left(x^2+1\right)^{5/2} dx$, let $u = x^2 + 1 \Rightarrow du = 2x\,dx$ and $x^2 = u - 1$. Then

$$I = \int x^2\left(x^2+1\right)^{5/2}(x\,dx) = \tfrac{1}{2}\int (u-1)\,u^{5/2}\,du = \tfrac{1}{2}\int\left(u^{7/2} - u^{5/2}\right)du = \tfrac{1}{2}\left(\tfrac{2}{9}u^{9/2} - \tfrac{2}{7}u^{7/2}\right) + C$$

$$= \tfrac{1}{9}\left(x^2+1\right)^{9/2} - \tfrac{1}{7}\left(x^2+1\right)^{7/2} + C.$$

4.3

40. $$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n^3}(2k+1)^2 = \lim_{n\to\infty}\frac{1}{n^3}\left[4\sum_{k=1}^{n}k^2 + 4\sum_{k=1}^{n}k + \sum_{k=1}^{n}1\right] = \lim_{n\to\infty}\frac{1}{n^3}\left[\frac{4n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} + n\right]$$

$$= \lim_{n\to\infty}\frac{2}{3}\left[\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 2\left(\frac{1}{n}+\frac{1}{n^2}\right) + \frac{1}{n^2}\right] = \frac{4}{3}$$

42. $$\lim_{n\to\infty}\sum_{k=1}^{n}\left[1 + 2\left(\frac{k}{n}\right)^2\right]\left(\frac{2}{n}\right) = \lim_{n\to\infty}\left(\frac{2}{n}\right)\left[\sum_{k=1}^{n}1 + \frac{2}{n^2}\sum_{k=1}^{n}k^2\right] = \lim_{n\to\infty}\left[\left(\frac{2}{n}\right)n + \frac{2}{n}\cdot\frac{2}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[2 + \frac{2}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = 2 + \frac{4}{3} = \frac{10}{3}$$

44. $$\lim_{n\to\infty}\sum_{k=1}^{n}\left(1 + \frac{2k-1}{2n}\right)\left(\frac{1}{n}\right) = \lim_{n\to\infty}\frac{1}{n}\left[\sum_{k=1}^{n}\frac{2n+2k-1}{2n}\right] = \lim_{n\to\infty}\frac{1}{2n^2}\left[(2n-1)\sum_{k=1}^{n}1 + 2\sum_{k=1}^{n}k\right]$$

$$= \lim_{n\to\infty}\left[\frac{2n-1}{2n^2}\cdot n + \frac{1}{n^2}\cdot\frac{n(n+1)}{2}\right] = \lim_{n\to\infty}\left[1 - \frac{1}{2n} + \frac{1}{2}\left(1+\frac{1}{n}\right)\right] = \frac{3}{2}$$